Hydrostatic Stress Effects in Low Cycle Fatigue

A Dissertation

by

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Outline

• Technical Background
• Mechanical Testing
• Constitutive Model Development
• Finite Element Model Development
• Finite Element Results
• Conclusions and Recommendations
Hydrostatic Stress Effects in Low Cycle Fatigue

Classical Metal Plasticity

- Bridgman’s experiments
- Yield – independent of hydrostatic or mean stress, $\sigma_m$
- Internal hydrostatic pressure – notched or cracked geometries

$$\sigma_m = \frac{1}{3} I_1 = \frac{1}{3} \left( \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \right)$$

$$p = -\sigma_m = -\frac{1}{3} I_1$$

- Determine three quantities:
  1. Yield Function
  2. Hardening Rule
  3. Flow Rule

Hydrostatic Pressure
Yield Function

- Yield function, $f$

$$f = f(\sigma_1, \sigma_2, \sigma_3)$$

- $f < 0$; Elastic material behavior

- $f = 0$; Yielding occurs, plastic material behavior

- Independent of hydrostatic stress
- Almost always used in computational plasticity
- Recommended for finite element analysis

von Mises yield function

$$f(J_2) = J_2 - k^2$$

$$\sigma_{eff} = \sqrt{3J_2}$$

$k$ – Yield strength in pure shear
Hydrostatic Stress Effects in Low Cycle Fatigue

Hardening Rule

Yield surface – change size, location, or both

Isotropic Hardening + Kinematic Hardening = Combined Hardening
Flow Rule

- Relates stresses and plastic strain increments
- Analogous to Hooke’s law for elastic behavior

**General flow rule**

\[ d\varepsilon_{ij}^{pl} = \frac{\partial g}{\partial \sigma_{ij}} d\phi \]

- \( g \) – Plastic potential function
- \( d\phi \) – Positive constant

**Associated flow, \( f = g \)**

\[ d\varepsilon_{ij}^{pl} = \frac{\partial f}{\partial \sigma_{ij}} d\phi \]
Hydrostatic Stress Effects in Low Cycle Fatigue

Richmond, Spitzig, and Sober’s Experiments

\[ \sigma_{\text{eff}} = (3J_2)^{1/2} \]

- \( \sigma_{\text{eff}} = \) Effective stress
- \( \sigma_c = \) Theoretical cohesive strength
- \( a = \) Slope
- \( d = \) Modified yield strength
- \( \sigma_{\text{eff}} = d - aI_1 \)

Yield Function
\[ f(I_1, J_2) = \sqrt{3J_2} + aI_1 - d \]
Hydrostatic Stress Effects in Low Cycle Fatigue

Drucker-Prager Yield Function

\[ f(I_1, J_2) = \sqrt{3J_2} + aI_1 - d \]

- Dependent on hydrostatic stress
- Often used in compacted soil calculations
- Seldom recommended for metal plasticity calculations
Hydrostatic Stress Effects in Low Cycle Fatigue

Low Cycle Fatigue

- Strain dependent material response
- Total Strain, \( \Delta \varepsilon = \Delta \varepsilon_e + \Delta \varepsilon_{pl} \)
  
  or \( \Delta \varepsilon/2 = \Delta \sigma/2E + \Delta \varepsilon_{pl}/2 \)

- Sum of elastic and plastic lines
- Coffin-Manson Equation,
  \[
  \frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c
  \]

Log Scale

\[\frac{\Delta \varepsilon}{2} \] vs \[\varepsilon'/E\]

Total = Elastic and Plastic

Elastic
Plastic
Hydrostatic Stress Effects in Low Cycle Fatigue

Low Cycle Fatigue Research

**Topics**
- Empirical formulas and modifications of Coffin-Manson equation
- Use of stress or strain invariants
- Use of the critical plane

**Researchers**
- Mowbray (1980)
- Kalluri and Bonacuse (1993)
- Lefebvre (1989)
- Brown and Miller (1973)
- Lohr and Ellison (1980)

Pressure dependent yield function in low cycle fatigue?
Hydrostatic Stress Effects in Low Cycle Fatigue

Research Program

Goal: Use FEA to simulate first few cycles of LCF tests using a pressure-dependent yield function

Experimental Program
• Two materials
  - 2024-T851
  - Inconel 100 (IN100)
• Material properties
• LCF test data

Analytical Program
• Pressure-dependent constitutive model with combined hardening
• Finite element models
• Compare FEA and experimental results

Hydrostatic stress effect in low cycle fatigue?
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Hydrostatic Stress Effects in Low Cycle Fatigue

Mechanical Tests

- Alignment
- Elastic constants
- Smooth uniaxial tensile
- Smooth uniaxial compression
- NRB tensile
- NRB low cycle fatigue
- Smooth round bar LCF

All per ASTM standards
2024-T851 Tension and Compression

- L direction
- $E - 5\%$ higher for compression
- 0.2\% offset $\sigma_y$ – approx. same
- L-T direction, similar results
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB Tension

- 6 variations of $\rho$
- 0.005 in. – 0.120 in.
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB Low Cycle Fatigue

- 3 variations of $\rho$, $\rho = 0.040$, 0.080, 0.120 in.
- Strain control
- $\pm 0.004$ in. gage displacement
- 0.040 in., $N_f \approx 10$ cycles
- 0.080 in., $N_f \approx 27$ cycles
- 0.120 in., $N_f \approx 44$ cycles

$\rho = 0.080$ in.
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 Smooth Round Bar LCF

- Strain control
- $\pm 0.015$ in. gage strain
- $N_f \approx 37$ cycles
- Specimen AD01 - interrupted after 10 cycles, then monotonically loaded to failure
- Transitional cyclic stress-strain curve
Outline

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Constitutive Model Development

Why?
• ABAQUS built-in models
  – Multilinear isotropic and bilinear kinematic hardening for von Mises
  – Only multilinear isotropic hardening for Drucker-Prager
• No linear combined hardening models

What?
• Pressure-dependent (Drucker-Prager) constitutive model
• Combined multilinear kinematic and multilinear isotropic hardening

How?
• ABAQUS user subroutine (UMAT) function
• Code written in FORTRAN
Hydrostatic Stress Effects in Low Cycle Fatigue

Drucker-Prager UMAT

• Yield function – \[ f = \sigma_{\text{eff}} - 3ap - \sigma_{ys} (\varepsilon_{eq}^{pl}) = 0 \]

Split plastic strain rate into 2 parts – \[ \dot{\varepsilon}_{ij}^{pl} = \dot{\varepsilon}_q \bar{n}_{ij} + \frac{1}{3} \dot{\varepsilon}_p \bar{I} \]

Numerical integration – backward Euler method, implicit

Incremental solution, 5 equations and 5 unknowns: \( p, \sigma_{\text{eff}}, \Delta \varepsilon_p, \Delta \varepsilon_q, \Delta \varepsilon_{eq}^{pl} \)
Hydrostatic Stress Effects in Low Cycle Fatigue

**Bilinear Hardening**

\[ H = \frac{EE_t}{E - E_t} \]

- Yield (effective) stress – linear function of equivalent plastic strain
- Commonly seen in literature and FEA code
Hydrostatic Stress Effects in Low Cycle Fatigue

Multilinear Hardening

- $H$ not a constant
- Flexibility in modeling complex stress-strain behaviors
- Possibly multiple values of $H$ for a given increment in plastic strain
- Yield stress – piecewise linear function of equivalent plastic strain
- Increases complexity of constitutive model equations
Outline

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• Finite Element Model Development
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Finite Element Analysis Overview

- Two constitutive models: von Mises and Drucker-Prager
- UMAT with combined multilinear hardening
- Large strain analyses
- Q4 elements with full integration
- Displacement control
- Symmetry utilized
- Three levels of mesh refinement: coarse, medium, fine
- Medium mesh used for all analyses
2024-T851 Material Property Inputs

- Elastic constants from mechanical test results
- Uniaxial tensile data – $\sigma - \varepsilon^{pl}$ table
- Power law to complete table
Notched Round Bar FEM

- Two symmetry planes
- Axisymmetric elements
- 6 notch geometries
Hydrostatic Stress Effects in Low Cycle Fatigue

Equal-Arm Bend FEM

• One symmetry plane
• Plane strain elements
• IN100
Hydrostatic Stress Effects in Low Cycle Fatigue

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• Technical Background
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UMAT Program Verification

- Check accuracy of elasto-plastic equations and corresponding FORTRAN code
- Compare UMAT with built-in ABAQUS models
- Compare solutions for different values of the combined hardening parameter, $\beta$
- Use NRB with $\rho = 0.040$ in. and smooth compression geometries
- Use 2024-T851 material properties
UMAT Program Verification

- NRB ($\rho = 0.040$ in.)
- Multilinear isotropic hardening
UMAT Program Verification

• NRB ($\rho = 0.040$ in.)
• Vary $\beta$ values
• Pure isotropic, $\beta = 1.0$
• Pure kinematic, $\beta = 0.0$
• Linear combination of kinematic and isotropic, $0.0 > \beta > 1.0$
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB ($\rho = 0.080$ in.) Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) First Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Fifth Cycle

![Graph showing gage displacement vs load with different loading cycles and material models.]
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Tenth Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

Equal-Arm Bend First Cycle

![Graph showing load vs. microstrain with data points and curves for Specimen Test Data, von Mises FEA, and Drucker-Prager FEA.]

Small strain analysis

Strain Gage

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Hydrostatic Stress Effects in Low Cycle Fatigue

Equal-Arm Bend Second Cycle

Small strain analysis
Hydrostatic Stress Effects in Low Cycle Fatigue

Equal-Arm Bend Third Cycle

Small strain analysis
Overall Conclusions

• Drucker-Prager solutions more accurately predicted the behavior of the test specimens for first few cycles

• Once the stable material response was reached, neither the Drucker-Prager nor von Mises results were entirely satisfactory

• Neither solution truly captures the shapes of the hysteresis loops
Supporting Conclusions

1. Careful experimental testing - a necessity when attempting to model a specimen’s LCF behavior
2. UMAT - useful as a building block for future studies in LCF behavior
3. Drucker-Prager constitutive model - superior to the von Mises model for simulating tensile monotonic test behavior
4. Drucker-Prager model - superior to the von Mises model for simulating the first few LCF cycles
5. Simulating the equal-arm bend three-cycle proof test - both models performed equally well
Recommendations

1. Develop new method to determine the Drucker-Prager constant, $a$.
2. Develop the pressure-dependent Jacobian.
3. Modify existing pressure-dependent UMAT - improve the modeling capability after the first few cycles.
4. Generate additional LCF data from notched specimens.
Hydrostatic Stress Effects in Low Cycle Fatigue

Additional Information

Extra Slides
Bridgman’s Experiments

<table>
<thead>
<tr>
<th>Early Tests (1930’s)</th>
<th>Later Tests (1940’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Smooth tensile bars</td>
<td>• More precise instrumentation</td>
</tr>
<tr>
<td>• Variety of metals</td>
<td>• Yield was dependent on hydrostatic pressure</td>
</tr>
<tr>
<td>• External hydrostatic pressures up to 450 ksi</td>
<td>• Not mentioned in plasticity textbooks</td>
</tr>
<tr>
<td>• No significant influence on yield until highest pressures</td>
<td></td>
</tr>
<tr>
<td>• Reported by early plasticity researchers</td>
<td></td>
</tr>
</tbody>
</table>
Richmond, Spitzig, and Sober’s Experiments

- Tests in the 1970’s and early 1980’s
- Tension and compression under hydrostatic pressure up to 160 ksi
- Maraging steel, HY-80, AISI 4310, 4330
- Grade 1100 Aluminum
- Polyethylene and polycarbonate
Recent Developments

- Pan, et al., Univ. of Michigan – 1996 - Present
- Lissenden, et al., Penn. State Univ. – 1999 – Present
- Wilson & Allen, Tenn. Tech. Univ. – 1997 - Present

Drucker-Prager yield theory to model pressure sensitivity of non-geological materials
## 2024-T851 Tension and Compression

### Tension

<table>
<thead>
<tr>
<th>Specimen Direction</th>
<th>Statistical Measure</th>
<th>Young's Modulus, $10^3$ ksi (GPa)</th>
<th>0.2% Offset Yield Strength, ksi (MPa)</th>
<th>Ultimate Tensile Strength, ksi (MPa)</th>
<th>True Fracture Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Average</td>
<td>10.6 (72.4)</td>
<td>67.8 (467)</td>
<td>76.6 (528)</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.05 (0.55)</td>
<td>0.84 (5.81)</td>
<td>0.55 (3.83)</td>
<td>0.003</td>
</tr>
<tr>
<td>L-T</td>
<td>Average</td>
<td>10.7 (73.6)</td>
<td>66.8 (460)</td>
<td>73.6 (507)</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.08 (0.55)</td>
<td>0.45 (3.13)</td>
<td>2.19 (15.34)</td>
<td>0.002</td>
</tr>
</tbody>
</table>

### Compression

<table>
<thead>
<tr>
<th>Specimen Direction</th>
<th>Statistical Measure</th>
<th>Young's Modulus, $10^3$ ksi (GPa)</th>
<th>0.2% Offset Yield Strength, ksi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Average</td>
<td>11.1 (76.2)</td>
<td>67.8 (467)</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.12 (0.84)</td>
<td>0.45 (2.68)</td>
</tr>
<tr>
<td>L-T</td>
<td>Average</td>
<td>11.3 (77.6)</td>
<td>67.8 (467)</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.20 (1.52)</td>
<td>0.45 (2.68)</td>
</tr>
</tbody>
</table>
# Hydrostatic Stress Effects in Low Cycle Fatigue

## IN100 Tension and Compression

### Tension

<table>
<thead>
<tr>
<th>Statistical Measure</th>
<th>Young's Modulus, $10^3$ ksi (GPa)</th>
<th>Upper Yield Strength, ksi (MPa)</th>
<th>$0.2%$ Offset Yield Strength, ksi (MPa)</th>
<th>Ultimate Tensile Strength, ksi (MPa)</th>
<th>True Fracture Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>31.8 (220)</td>
<td>172 (1184)</td>
<td>167 (1150)</td>
<td>235 (1618)</td>
<td>0.230</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.32 (1.99)</td>
<td>1.49 (10.42)</td>
<td>2.96 (20.26)</td>
<td>0.82 (5.72)</td>
<td>0.010</td>
</tr>
</tbody>
</table>

### Compression

<table>
<thead>
<tr>
<th>Statistical Measure</th>
<th>Young's Modulus, $10^3$ ksi (GPa)</th>
<th>Upper Yield Strength, ksi (MPa)</th>
<th>$0.2%$ Offset Yield Strength, ksi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>32.6 (225)</td>
<td>172 (1185)</td>
<td>167 (1153)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.29 (2.17)</td>
<td>0.41 (2.86)</td>
<td>0.52 (3.61)</td>
</tr>
</tbody>
</table>
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) First Cycle

[Diagram showing gage displacement vs. load for different specimens]
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Second Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Third Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Fourth Cycle

![Graph showing gage displacement and load for different specimens in the fourth cycle.](image)
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Fifth Cycle

Load, $P$ (lbs)

Gage Displacement, $v$ (in.)

30th Cycle, $\rho = 0.120$ in.

- Specimen 122
- Specimen 123
- Specimen 124
- Specimen 125
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Tenth Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Twentieth Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.120$ in.) Thirtieth Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

IN100 Tension and Compression

- $E$ – 2.5% higher for compression
- 0.2% offset $\sigma_y$ – approx. same
Hydrostatic Stress Effects in Low Cycle Fatigue

IN100 Low Cycle Fatigue

- Equal-arm bend specimen
- Three-cycle proof test
- 3% strain to 1.3% strain
- Conducted by Pratt and Whitney
Hydrostatic Stress Effects in Low Cycle Fatigue

IN100 Material Property Inputs

- Elastic constants from mechanical test results
- Uniaxial tensile data – $\sigma - \varepsilon_{pl}$ table
- Linear fit to complete table

![Graph showing true stress vs. plastic strain for IN100 material properties.](image)
Smooth Tensile Bar FEM

- Two symmetry planes
- Axisymmetric elements
- 0.25 in. and 0.35 in. diameter tensile bars modeled
Smooth Compression Cylinder FEM

- Two symmetry planes
- Axisymmetric elements
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.005\text{in.}$) Mesh Convergence

Effective stress across the neck at maximum load
NRB ($\rho = 0.005\text{in.}$) Mesh Convergence

Mean stress across the neck at maximum load
NRB ($\rho = 0.005\text{in.}$) Mesh Convergence

Radial stress across the neck at maximum load
NRB ($\rho = 0.005$in.) Mesh Convergence

Equivalent plastic strain across the neck at maximum load
UMAT Program Verification

- NRB ($\rho = 0.040$ in.)
- Bilinear Kinematic hardening
UMAT Program Verification

- Smooth Compression Cylinder
- Vary $\beta$ values
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 Smooth Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 Smooth Compression
2024-T851 NRB ($\rho = 0.005$ in.) Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB ($\rho = 0.010$ in.) Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB ($\rho = 0.020$ in.) Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB ($\rho = 0.040$ in.) Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

2024-T851 NRB ($\rho = 0.120$ in.) Tensile

![Graph showing Gage Displacement vs Load]
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB (\( \rho = 0.040 \text{ in.} \)) First Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.040$ in.) Third Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.040$ in.) Ninth Cycle
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.080$ in.) First Cycle

![Graph showing gage displacement and load for NRB (\(\rho = 0.080\) in.) First Cycle, comparing experimental data with theoretical predictions.](image-url)
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.080$ in.) Fifth Cycle

![Graph showing gage displacement vs. load for NRB (\(\rho = 0.080\) in.) Fifth Cycle.](image)
Hydrostatic Stress Effects in Low Cycle Fatigue

NRB ($\rho = 0.080$ in.) Tenth Cycle

![Graph showing gage displacement and load for NRB specimen 803, von Mises FEA, and Drucker-Prager FEA models during the tenth cycle. The graph displays data points and curves for each model, indicating differences in stress behavior.]
IN100 Smooth Tensile
Hydrostatic Stress Effects in Low Cycle Fatigue

IN100 Smooth Compression

![Graph showing load displacement for IN100 Smooth Compression](image-url)
Large strain analysis
Hydrostatic Stress Effects in Low Cycle Fatigue

Equal-Arm Bend Second Cycle

![Graph showing load versus microstrain for specimen test data, von Mises FEA, and Drucker-Prager FEA. The graph indicates large strain analysis.](image)

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Hydrostatic Stress Effects in Low Cycle Fatigue

Equal-Arm Bend Third Cycle

Large strain analysis
Supporting Conclusions II

1. Careful experimental testing - a necessity when attempting to model a specimen’s LCF behavior
2. UMAT - useful as a building block for future studies in LCF behavior
3. Drucker-Prager constitutive model - superior to the von Mises model for simulating tensile monotonic test behavior
4. Simulating monotonic compressive behavior - unclear which model is superior
5. Drucker-Prager model - superior to the von Mises model for simulating the first few LCF cycles
6. Simulating the equal-arm bend three-cycle fatigue test - both models performed equally well