Solving Systems of Linear Equations in MATLAB

Mike Renfro

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Example 3.27

Find the solution to the following system of equations:

\[
\begin{bmatrix}
-4 & 1 & 1 & 0 \\
1 & -4 & 0 & 1 \\
1 & 0 & -4 & 1 \\
0 & 1 & 1 & -4 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
=
\begin{bmatrix}
-200 \\
-400 \\
0 \\
-200 \\
\end{bmatrix}
\]
MATLAB Setup

\[
A = \begin{bmatrix}
-4 & 1 & 1 & 0 \\
1 & -4 & 0 & 1 \\
1 & 0 & -4 & 1 \\
0 & 1 & 1 & -4
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
-200 \\
-400 \\
0 \\
-200
\end{bmatrix}
\]
2 MATLAB Solutions

>> x = inv(A)*b

x =
100.0000
150.0000
50.0000
100.0000

>> x = A\b

x =
100.0000
150.0000
50.0000
100.0000

Both of these methods work fine for this problem, but in general, we’ll favor using A\b over inv(A)*b for both speed and accuracy reasons.
Example 3.29 (Corrected)

Use MATLAB to find the inverse of the matrix $[A]$ given by

$$[A] = \begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{3} & \ldots & \frac{1}{n} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \ldots & \frac{1}{n+1} \\
\frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \ldots & \frac{1}{n+2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{1}{n} & \frac{1}{n+1} & \frac{1}{n+2} & \ldots & \frac{1}{2n-1}
\end{bmatrix}, \quad n = 50$$

and find the error in the inverse matrix by calculating $[C] = [A][A]^{-1}$ and summing up the absolute values of the matrix elements. If there was no error at all, the sum would be $n$ (it’s not the best test of error, but if the sum isn’t $n$, we know we’ve got substantial error somewhere).
MATLAB Solution 1

```matlab
>> A = hilb(50);
>> result = inv(A)*A;
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 1.449122e-021.
>> err = sum(abs(result(:)))
err =
   4.4724e+004
```
>> A = hilb(50);
>> result = A \ A;
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 1.603366e-020.
>> err = sum(abs(result(:)))
err =
    3.6850e+003
>>

The error on this version is much, much lower. This is why we don't use the \() function on ill-conditioned matrices.
Comparison of Error

Using $A \backslash A$

Using $\text{inv}(A) \ast A$