A Novel High-Resolution Algorithm for Complex-Direction Search

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ABSTRACT
The location dependent effect of acoustic field received by a linear array is modeled as an extra “loss” factor in the complex spectral variable. A high-resolution algorithm combining the Singular Value Decomposition Method and the Eigen-Matrix Pencil Method is then employed to find the complex directions representing the incoming directions and the location dependent factors of multipath and multimode arrivals. Five key features (namely, noise immunity, robustness, resolution, accuracy, and physical insight) of the proposed algorithm are studied using numerical examples.

1. INTRODUCTION
A linear receiving array is commonly employed to discern and to evaluate the arriving angles of multi-path arrivals. In the conventional approaches, the total field \( P \) of multipath or multimode arrivals received by the linear array is first expressed in terms of a sum of exponentials:

\[
P(x) = \sum_{n=1}^{N} A_n \exp(iK_n x) + W(x)
\]  

(1)

where \( K_n \) and \( A_n \) are spectrum and strength, respectively, of the \( n \)th arrival, \( W \) represents the background noise, and \( x \) denotes the coordinates of receivers along the linear array. Various spectral estimation methods can then be used to estimate \( K_n \) and \( A_n \) of relevant arrivals, and the arriving directions will be derived from the estimated \( K_n \). However, there are two deficiencies in those conventional approaches. First, the amplitude \( A_n \) is assumed to be \( x \)-independent. This assumption is usually not very practical because the gains of different receivers may not be well calibrated, the locations of these receivers may not lie along a straight line, the incoming waves may be beam-like fields, the incoming wavefronts may not be on plain surfaces, or there may be more than two waves arriving with very close incoming directions but not totally in phase, etc. All these practical factors can cause \( A_n \) to be dependent on \( x \). Second, the spectrum \( K_n \) is assumed to be real. This is not always true. For example, consider the case of a horizontal array in an underwater waveguide where \( P \) denotes the total acoustic pressure, \( x \) is the range coordinate, and \( K_n \) and \( A_n \) are the modal eigenvalues and amplitudes, respectively. If the waveguide is lossy, the modal eigenvalue \( K_n \) will be complex. To remedy these two deficiencies, we first extend the conventional model to incorporate the \( x \)-dependent effects and the loss mechanism into the model. Let \( A_n = c f_n(x) \) and \( K_n = -i \alpha_n + \beta_n \) where \( \alpha_n \) accounts for the loss mechanisms and \( f_n(x) \) accounts for the \( x \) dependency of \( A_n \). Noting that \( f_n = \exp(\ln(f_n(x))) \) and \( \ln(1+\varepsilon) = \varepsilon \) if \( \varepsilon << 1 \) and choosing a reference coordinate \( x_0 \), the received field \( P \) can be expressed in the same form as the above equation:

\[
P(x) = \sum_{n=1}^{N} a_n \exp(ik_n x) + W(x) ,
\]

\[
k_n = \beta_n - i \alpha_n , \quad a_n = c_n f_n(x_0) \exp(-\alpha_n x_0) ,
\]

\[
\alpha_n = \alpha_n + \alpha_n , \quad \alpha_n = [\partial f_n(x_0)/\partial x_0] f_n(x_0)
\]  

(2)

2. METHODOLOGY
Having set up the model, we can then use a direction-resolving algorithm to evaluate the complex amplitudes \( a_n \) and the complex exponents \( ik_n \) if the algorithm has good characteristics in the
following five key issues: a) noise immunity; b) robustness; c) resolution; d) accuracy; and e) relation between estimated results and physical mechanisms. Although there exist many conventional approaches, none can work satisfactorily. The spectral resolution of the FFT-based approaches is limited by the array length which is relatively fixed by the experimental setup. Periodogram, Black-Tukey, and Multiple Signal Classification methods cannot provide the real parts $\sigma_n$ of the complex exponents. Prony-based algorithms can only work in conditions with very low signal to noise ratios. The AutoRegressive and AutoRegressive Moving Average series usually have trouble in relating their estimated results directly to the physical unknowns $a_n$ and $k_n$. To remedy these difficulties, we apply a newly developed modified eigen-matrix pencil which is originally developed for identifying scattering centers for target identification [1]. This new algorithm consists of three steps. First, the Singular Value Decomposition (SVD) method [2] is employed to filter out the white noise. Secondly the Eigen-Matrix Pencil Method [3,4] is used to identify the complex damped exponent. Finally, the complex amplitudes can be obtained by a Least Square approach. The Cramer-Rao bound is used as a benchmark in the numerical simulations where our method is proven to be most successful for the current applications. The procedure of implementing this algorithm is summarized as the following:

1. Form the Hankel matrix $H$ using the measured noisy data sequence $x[m]$, $m=0,1,\ldots,2M+1$. Note that $M>N$.

2. The number of signals $N$ (see eqs. (1) or (2)) is usually not known a priori and is estimated by the SVD expansion of the matrix $H$. Keep only the $N$ principal components and build a new matrix $H_n$ using only the $N$ singular values and their associated eigenvectors.

3. Find the complex eigenvalues $\lambda_n$ and associated eigenvectors $e_n$ of the matrix $H_n$, $n=1,2,\ldots,M+1$. There are $N$ dominant eigenvalues.

4. Form the eigen-matrix $E_1 = \{ e_1, e_2, \ldots, e_N \}$. Then form the eigen-matrix $F_1$ using the first $L$ rows of $E_1$, ranging from the first row to the $L$-th row of $E_1$, and another eigen-matrix $F_L$ using the next $L$ rows of $E_1$, ranging from the second row to the $(L+1)$-th row of $E_1$. Note that $L>N$.

5. Form the eigen-matrix pencil $\{ F_1^H F_1, F_1^H F_L \}$ and find its complex generalized eigenvalues $\lambda_n, n=1,2,\ldots,N$.

6. Obtain the $N$ signal poles using $z_n = z_n^{-1} = \exp[(\alpha_n + j2\pi f_n)X], n=1,2,\ldots,N$, where $X$ is the sampling interval.

7. Obtain the residues $a_k, k=1,2,\ldots,N$ using the least square fitting the $(2M+1)$ data.

3. Numerical Results
In our previous paper [5], we have successfully applied this method to model broadband wireless multipath channels. Here, our approach is employed to estimate the complex directions of eight rays, among which three rays have location dependencies and others have no location dependencies. Fig. 1a shows the received field as a function of receiver location. Figs. 1b and 1c show the estimated spectra derived by using the FFT with a Hamming window and our method, respectively. It is interesting to note the FFT is not able to resolve the rays correctly, but our approach is able to provide good resolution. In addition to providing accurate results of the spectra of the channel, our approach can also supply the location dependencies of the rays, which is not shown here. The combination of the real spectral number $\beta_m$ amplitude $a_m$, and the location dependence $\alpha_n$ can provide us more physical insights into the multipath or multimode arrivals.

To further characterize the performance of this approach statistically, we consider a two-ray model with additive white noise in this paper. In Fig. 2, we made 50 independent Monte Carlo runs to obtain 50 independent shots of each pole on the complex $Z$ plane, where an independent Gaussian noise series is generated in each run. We notice that there are no spurious poles in our method. Moreover, we can see from Fig. 2 that the mean estimated value is close to the "true" values and the standard deviation or variance of the estimated values is small. (This is called "robust"). This means that our algorithm is very accurate and robust, since for almost every shot the estimated poles hit the right positions on the $Z$ plane.

The variance of any unbiased algorithm is bounded by the Cramer-Rao (CR) lower bound. Such a bound is very useful because it tells the best possible performance (smallest variances) for an unbiased estimator. Estimators or algorithms
whose variance is close to or equals this bound can then be said to be "optimal". In Figs.3-4, we made 500 independent Monte Carlo runs to obtain the mean and the variance where an independent Gaussian noise is generated in each run. Fig.3 shows the mean values and Fig.4 shows the variance. Parts (a) and (b) of both figures report the outcomes of $k_n$ and $\alpha_n$, respectively. The daggered curve and the dotted curve represent the estimated values of the first ray and second ray, respectively. Solid lines in Fig.3 are the true values and solid lines in Fig.4 represent the Cramer-Rao (CR) lower bounds. Fig.3 shows that our algorithm yields good results for SNR>10 dB. (Without using the SVD to filter out the noise, the algorithm only works when SNR>30 dB.) Since the variance in Fig. 4 is so close to the CR bound, our algorithm is almost "optimal" in the sense of the smallest variance for SNR>10dB. But it is unbiased only up to SNR=22dB. Strictly speaking, the CR bound applies only to an unbiased estimator. However, it is still a good reference for our algorithm which is with small bias. It is shown from this example that the proposed method has good noise immunity features.

4. Conclusions

Complex spectra can be used to model the location dependent features of a linear array for a direction search of multiple arrivals. A combination of the Singular Value Decomposition method and the Eigen-Matrix Pencil Method is proven to be very useful for finding the complex directions.

References


Fig.1 Estimation of a 8-arrival model: a) the received field as a function of receiver location; b) the estimated spectra derived by using the FFT with a Hamming window; and c) the estimated spectra derived by using our method.
Fig. 2 Estimation of the complex directions of a two-ray model in the complex $Z$-plane.

Fig. 3a Mean values of $k_w$. The daggered curve and the dotted curve represent the estimated values of the first ray and second ray, respectively. Solid lines the true values.

Fig. 3b Mean values of $\alpha_w$. The daggered curve and the dotted curve represent the estimated values of the first ray and second ray, respectively. Solid lines the true values.

Fig. 4a Variance of $k_w$. The daggered curve and the dotted curve represent the estimated values of the first ray and second ray, respectively. Solid lines in represent the Cramer-Rao (CR) lower bounds.

Fig. 4b Variance of $\alpha_w$. The daggered curve and the dotted curve represent the estimated values of the first ray and second ray, respectively. Solid lines in represent the Cramer-Rao (CR) lower bound.