Detection of Physics-based Ultra-wideband Signals Using Generalized RAKE in Presence of Inter-Symbol Interference

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Abstract—The generalized rake receiver based on physics-based channel model is proposed for UWB outdoor application to estimate and compensate for the pulse distortion. The successive channel estimation is adopted to generate the per-path waveform as the template for the generalized rake receiver. The receiver can eliminate the effect of pulse distortion appearing in a lot of UWB channels. MMSE linear equalizer is used to handle intersymbol interference (ISI) for achieving high data rate.

Index Terms—Generalized rake receiver, Physics-based channel, Successive channel estimation, Equalization and Ultra-wideband (UWB)

I. INTRODUCTION

Ultra-wideband (UWB) communications involves the transmission of short pulses with a relatively fractional bandwidth larger than 20 % or a 10-dB bandwidth exceeding 500MHz. Its high bandwidth and potential capacity make it an attractive candidate technology for home area communication and sensor networks. Most research on UWB communication is based on impulse radio [1-3]. When a short pulse propagates through a channel, multiple pulses are received via multipath. These received pulses in general have pulse shapes different from the incident short pulse for UWB case [3-5]. The unique phenomenon is called pulse distortion caused by frequency dependency of the propagation channel and antennas. This problem has become practically significant after the concept of frequency dependency was adopted in the IEEE 802.15.4a [7]. The impact of pulse distortion on the baseband transmission has been investigated using a matched filter in the receiver front-end [4-6]. It is demonstrated that pulse distortion can greatly degrade the system performance if no compensation is carried out. In [8] the inter-symbol-interference (ISI) is neglected. One wonder if pulse distortion degrades system performance differently when ISI is present. To answer this question is the central result in this paper.

In the present paper a two-dimensional tap-delayed line model is introduced to model the UWB channel based on physics-based channel model and successive channel (SC) estimation. An FIR filter is used to present the channel per-path impulse response. Based on two-dimensional tap-delayed line model a generalized rake receiver is proposed to deal with per-path pulse distortion. The performance of the generalized rake receiver for single user is studied for both of ISI and No ISI case in the high-rise building environment. The generalized rake receiver has been extended to multiuser case and multiuser detection is used [8]. The paper is organized as following: Physics-based channel model is introduced in Section II. Section III presents the generalized rake receiver structure and UWB signal detection. The algorithms for successive channel estimation and equalizer coefficients estimation are given in Section IV and Section V, respectively. The numerical results are shown in Section VI. Section VII concludes the paper based on the results from previous sections.

II. PHYSICS-BASED CHANNEL MODEL

A big challenge of UWB is channel modeling where pulse distortion complicates system analysis. Mathematically the generalized channel model is expressed by [4-6]

$$h(\tau) = \sum_{n=1}^{P} A_n h_n(\tau) \ast \delta(\tau - \tau_n)$$ (1)

where P generalized paths are associated with amplitude $A_n$, delay $\tau_n$, and per-path impulse response $h_n(\tau)$. The $h_n(\tau)$ represents an arbitrary function that has finite energy. Symbol “$\ast$" denotes convolution and $\delta(x)$ is the Dirac Delta function. Turin’s model widely used for narrowband channels and some UWB channels is a special case of Eq. (1) if $h_n(\tau) = \delta(\tau), \forall n$. A large category of UWB signals can be expressed as

$$h_n(\tau) = \frac{A_n}{\Gamma(-\alpha_n)} \tau^{-(1+\alpha_n)} U(\tau), \quad H_n(\omega) = (j\omega)^{\alpha_n}$$ (2)

where $\alpha_n$ assumes an arbitrary real value, and $U(\cdot)$ and $\Gamma(\cdot)$ are the Gamma function and the unit function, respectively. Given for a special case for $\alpha_n = \alpha, \forall n$, this model results in the frequency dependency model recently accepted in IEEE 802.15.4a [7].

FIR filter can be used to represent the per-path impulse response $h_n(\tau)$ in Eq. (1):

$$h_n(\tau) = \sum_{m=1}^{M} \beta_{mn} \delta(\tau - \tau_{mn})$$ (3)

where the FIR filter is assumed to have M taps with tap spacing $T_s$. The received signal is sampled every $T_s$ seconds. Consequently, the two-dimensional tap-delayed line channel model is obtained and is rewritten as

$$h(\tau) = \sum_{n=1}^{P} \sum_{m=1}^{M} \tilde{a}_{mn} \delta(\tau - \tilde{\tau}_{mn})$$ (4)

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where $\tilde{a}_{mn} = A_n \beta_{mn}$ is the real amplitude of each tap corresponding to $\tilde{\tau}_{mn}$. With a mapping, the two-dimensional model is reduced to a one-dimensional discrete model

$$h(\tau) = \sum_{l=1}^{L} a_l \delta(\tau - \tau_l) \quad (5)$$

where

$$L = MP,$$

$$\tau_l = \tau_{[m+(n-1)M]} = \tilde{\tau}_{mn},$$

$$a_l = \tilde{a}_{mn} = A_n \beta_{mn},$$

$$l = m + (n-1)M, \quad m = 0, 1, \ldots, M, \quad n = 0, 1, \ldots, P.$$

The one-dimensional discrete model makes the channel estimation algorithms used for conventional rake receiver applicable to the generalized rake receiver. So a lot of channel estimation algorithms can be used for the generalized rake receiver.

### III. Signal Detection

Based on above two-dimensional tap-delayed line channel model, a generalized rake receiver is proposed in Fig.1 to compensate for per-path pulse distortion. It consists of a front-end filter and a bank of filters associated with delay lines. The front-end filter is matched to the transmitted pulse, and obtain the discrete output

$$s(t) = \sum_{k=0}^{\infty} b_k p(t - kT_s), \quad (7)$$

where $T_s$ is the symbol duration.

#### B. Received signal

The received signal $r(t)$ at front end of receiver can be expressed by

$$r(t) = h(t) * s(t) + n(t), \quad (8)$$

where $h(t)$ is the channel impulse response, $n(t)$ is white Gaussian noise with zero mean and variance $N_0/2$.

#### C. Generalized rake receiver

The output of the matched filter is expressed by

$$x(t) = r(t) * p(-t), \quad (9)$$

where $p(-t)$ is the time reversal of $p(t)$, the transmitted pulse.

After matched filter, we sample $x(t)$ at sampling rate $1/\Delta$ and obtain the discrete output $x[n]$.

$$x[n] = x(n\Delta), \quad (10)$$

where $\Delta$ is chosen to be less than the minimum time difference among rake fingers.

The output of the $k^{th}$ FIR filter $y_k[n]$ is expressed by

$$y_k[n] = x[n] * h_k[n]$$

$$= \sum_{i=1}^{M} \alpha_{ki} x[n - \tau_{ki}], \quad (11)$$
where $h_k[n]$ is the impulse response of the $k^{th}$ FIR filter. 

After combining, the output $y[n]$ can be expressed by

$$y[n] = \sum_{i=1}^{P} y_i[n - T_i],$$  \hspace{1cm} (12)

where $T_i$ is the delay on the $i^{th}$ branch, and it is to equal to $\tau_{1i}$ defined in Eq.(4). The rake combining is implemented by a bank of FIR filters. The rake combining used in the paper is equivalent to Equal Gain combining(EGC). The parameters of FIR filters and delay lines can be obtained by channel estimation.

**D. Channel equalization**

A linear equalizer with $2N + 1$ taps is used to handle the intersymbol interference. The output of the equalizer $b[n]$ is

$$\hat{b}[n] = \sum_{k=-N}^{N} C_k y[n - k]$$  \hspace{1cm} (13)

where $C_k$ is the equalizer coefficients.

After thresholding the decision $b[n]$ is made by

$$\hat{b}[n] = \text{sign}(\hat{b}[n]).$$  \hspace{1cm} (14)

**IV. Channel estimation**

All rake receivers require knowledge of the channel parameters in order to detect properly the signal. The channel must be estimated prior to the actual detection. We use a data-aided (DA) approach [9,10] where the data frame begins with a pilot signal sequence $\{b^p\}$ consisting of $N_p$ known pilot symbols. The received signal $r$ is defined in Eq.(8). The covariance matrix $C$ has terms of noise variance on its diagonal and zeros elsewhere. In this paper the successive channel estimation is used for the one-dimensional discrete channel model. The estimated delay and amplitude are

$$\tau = \arg \max \left\{ \left| \xi(\tau) C^{-1} y(\tau) \right|^2 \right\}$$  \hspace{1cm} (15)

$$\hat{a} = \frac{\xi(\hat{\tau}) C^{-1} r}{\xi'(\hat{\tau}) C^{-1} r}$$  \hspace{1cm} (16)

where

$$[\xi(\tau)]_m = \sum_{i=0}^{N_p-1} b^p(i) r(mT_s - iT - \tau), \hspace{1cm} 1 \leq m \leq M.$$  

Here $p(t)$ with duration $T$ is the transmitted pulse. The above scheme can be performed iteratively for the multipath channel defined in (5). The algorithm is summarized by the following four steps in [10], originally in [9]:

1. Initialization: set threshold and $c(\tau) = 0$ for $\tau_{\min} \leq \tau \leq \tau_{\max}$.

2. Perform the search for the strongest tap $\hat{\tau}$ and calculate $\hat{\alpha}$ by using the above equations,

$$c(\hat{\tau}) \leftarrow c(\hat{\tau}) + \hat{\alpha}$$  

$$r \leftarrow r - \hat{\alpha} \xi(\hat{\tau});$$

3. If $\hat{\alpha} \geq \text{threshold}$, go to step 2; otherwise set $\hat{h}(\tau) = c(\tau)$ and stop.

Using the above successive channel estimation algorithm the channel impulse response $\hat{h}(\tau)$ is obtained.

With (3) and (4), the FIR representation of the per-path impulse response is estimated as $h_n(\tau)$. When the pulse waveform $p(\tau)$ is transmitted, the estimated received signal is $\hat{p}(\tau) \ast \hat{h}(\tau)$. For the $n^{th}$ path, the pulse waveform is $q_n(\tau) = p(\tau) \ast h_n(\tau)$. If one tap is used in Eq. (3), the generalized rake receiver is just the conventional rake receiver used in narrowband and UWB scenario. If several taps (say $M$) are used in Eq. (3), then $\hat{h}_n(\tau) = \sum_{m=1}^{M} \beta_{mn} \delta(\tau - \tau_{mn})$ and thus the received pulse waveform for the $n^{th}$ path is estimated as

$$\hat{q}_n(\tau) = \sum_{m=1}^{M} \beta_{mn} p(\tau - \tau_{mn}).$$  \hspace{1cm} (17)

The generalized rake receiver is designed to match $\hat{q}_n(\tau)$, instead of $p(\tau)$. In other words, the impulse response of the $n^{th}$ FIR filter is equal to $\hat{h}_n(\tau)$.

**V. Equalizer coefficients estimation**

After channel estimation, the output of generalized rake receiver $y^p$ can be expressed by

$$y^p[n] = \sum_{i=0}^{P} y_i^p[n - T_i],$$  \hspace{1cm} (18)

for $0 \leq n \leq N_p - 1$. For a MMSE equalizer with $2M + 1$ taps, by minimizing mean square error

$$\frac{1}{N_p} \sum_{n=0}^{N_p-1} \left[ \sum_{k=-M}^{M} C_k y^p[n - k] - b^p[n] \right]^2,$$  \hspace{1cm} (19)

the coefficients of the MMSE equalizer are given by

$$C = R_r^{-1} R_{xr},$$  \hspace{1cm} (20)

where $R_c$ and $R_{xr}$ are correlation matrix and vector, respectively, defined as

$$R_c = \begin{bmatrix}
R_c(0) & \cdots & R_c(M) \\
R_c(-1) & \cdots & R_c(M - 1) \\
\vdots & \ddots & \vdots \\
R_c(-2M) & \cdots & R_c(-M) \\
\end{bmatrix}$$

and

$$R_{xr} = [R_{xr}(-M) \cdots R_{xr}(-1), R_{xr}(0), R_{xr}(1) \cdots R_{xr}(M)]^T$$  \hspace{1cm} (21)

where

$$R_c(k - l) = \frac{1}{N_p} \sum_{n=0}^{N_p-1} y^p[n - l] y^p[n - k]$$  \hspace{1cm} (23)

and

$$R_{xr}(k) = \frac{1}{N_p} \sum_{n=0}^{N_p-1} b^p[n] y^p[n - k].$$  \hspace{1cm} (24)
VI. NUMERICAL RESULTS

In this paper we will investigate a general case of Eq.(1) where pulse distortion for two received paths is different. Since the statistical model is currently not available, a physics-based channel model is adopted. As an example, the high-rise building environment where multiple diffraction causes a severe pulse distortion is studied in the present paper. The propagation environment illustrated in Fig.3 can be represented by a channel model in a general form of Eq. (1). The detailed formulation and simulation of the channel modeling are reported in [5]. The channel has two paths. The second order Gaussian pulse with width 0.4 ns is used as the transmitted pulse. Two received pulse waveforms due to two different paths, namely \( q_1(\tau) \) and \( q_2(\tau) \), are quite different from that of the transmitted pulse shown in Fig.4.

In our simulation the sampling frequency is 80 GHz. The transmitted pulse waveform is chosen as the template for the successive channel estimation algorithm that is detailed in [8]. Although there can be many ways to select a template for the SC algorithm and even multiple templates can be employed, the general criterion is to choose a template waveform which has high similarity with the received pulse waveforms. The threshold for this algorithm is 30 dB down from the maximum amplitude. To achieve an energy capture loss of less than 4% at around \( E_b/N_0 = 5 \) dB, if 80 taps are used in the FIR representation of pulse distortion, a total of 512 pilot symbols is found to be sufficient in using the successive channel estimation.

Shown in Fig. 5 is the impact of the number of terms on the FIR representation of the first received pulses \( q_1(\tau) \), where \( q_1(\tau) \) is plotted in absence of noise. It is observed that the number of taps has visible impact on pulse representation, and the distorted pulse at the receiver can be represented by the 3 strongest terms in Eq. (3) to achieve good fitting accuracy in term of mean squared error.

Illustrated in Fig. 6 is the performance comparison in case of no ISI, where the data rate is 35.1Mbps. The performance of the generalized RAKE using different number of taps are bounded by the matched filter bound and conventional rake bound. The matched filter bound is obtained with perfect channel estimation in Eq.(1). The conventional rake bound is obtained where the received filter matches the input pulse waveform, which implies that the pulse distortion is not considered. The performance of the generalized rake with 80 taps is close to the matched filter bound. The curve of conventional rake (one tap representation) agrees very well with its theoretical curve and is 1.3dB away from that of the matched filter bound at \( BER = 10^{-3} \). The generalized rake with 3 taps representation is 1.1dB better than the conventional rake.

Fig. 7 illustrates the performance of the generalized rake receiver with and without equalizer in case of ISI. The data rate is 40Mbps. The same MMSE equalizer with 25 taps is used for both of the generalized rake and the conventional rake receiver. It is observed that the curve of the generalized rake with 3 taps representation is 5.6dB away from the matched filter bound and is 1.0 dB better than that of the
conventional rake at $BER = 10^{-3}$. The equalizer is necessary for both of the generalized rake and the conventional rake receiver if ISI occurs.

VII. CONCLUSION

In the present paper a generalized rake receiver based on the two-dimensional tap-delayed line channel model is investigated for a high-rise building environment. The successive channel estimation algorithm is adopted to generate the per-path waveform as the template for the rake receiver in order to compensate for the effect of pulse distortion appearing in a lot of UWB channels. The generalized rake receiver can significantly improve the system performance with and without ISI. Pulse distortion is founded to have similar impact on the performance of generalized RAKE with and without ISI. Another technique called time reversal mirror can be used to compensate for pulse distortion [8], compress the multipath delay spread, and thus reduce ISI. The scheme of time reversal mirror reduces the complexity of receiver, and an equalizer may not be necessary for high speed data transmission.

REFERENCES