Wireless Tomography in Noisy Environments using Machine Learning

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Abstract

This paper, one in a continuing series, describes a new initiative in wireless tomography. Our goal is to combine two technologies: wireless communication and radio frequency (RF) tomography, for the close-in remote sensing. The hybrid system including wireless communication devices for wireless tomography is proposed in this paper. Noise reduction, modified standard phase reconstruction, and imaging are exploited sequentially to perform wireless tomography in noisy environments. The performance given in this paper illustrates the significance and prospect of wireless tomography. The contributions of this paper are threefold: (1) the hybrid system provides a strong and flexible infrastructure for wireless tomography; (2) machine learning, especially non-linear dimensionality reduction, is explored to execute noise reduction and combat the non-linear noise effect; (3) modified standard phase reconstruction is well achieved using the de-noised amplitude-only total fields from the simple sensors and the received accurate full-data total fields from the advanced sensors. Experimental data provided by the Institute Fresnel in Marseille, France are used to demonstrate the concept of wireless tomography and validate the corresponding algorithms.

Index Terms

wireless tomography, noise reduction, machine learning, phase reconstruction, imaging

I. INTRODUCTION

As smart phones are widely used, there will be a potential large-scale wireless communication network deployed around the world [1]. Currently, we are interested in embedding remote sensing into this kind of large-scale wireless communication network. A new project called cognitive radio network as sensors has been launched [2]–[6]. However, wireless communication devices, components, and nodes are not specifically designed for remote sensing [1]. They do not meet the high-accuracy
measurement requirement of remote sensing. For example, radio frequency (RF) tomography or inverse scattering requires accurate phase information to perform imaging. Accurate phase information is hard to obtain using wireless communication devices, especially in the presence of noise [1].

Wireless tomography was first proposed in [7]. Wireless tomography, which combines wireless communication and RF tomography, gives a novel approach to remote sensing. Incoherent tomography, coherent tomography, and self-coherent tomography have been summarized and compared in [7]. Machine learning and waveform diversity have been explored in the context of wireless tomography to improve the system’s performance [8]. Potential applications of time reversal and compressive sensing to wireless tomography have been discussed in [7], [8].

When wireless communication devices instead of sophisticated equipment are exploited to perform wireless tomography, accurate phase information of the received total fields is hard to obtain. Thus, self-coherent tomography is proposed, which has two steps [7]. First, phase reconstruction is achieved using the received amplitude-only total fields. Second, the standard inverse scattering algorithms, e.g., Born iterative method, distorted Born iterative method, contrast source inversion [9]–[12], and so on, are used for data analysis and imaging.

However, the complex working situation and dynamic radio environment still cause many challenges for wireless tomography. For real-world applications, the effect of noise cannot be ignored. Previous literature on RF tomography or inverse scattering rarely addressed the noise issue. The corresponding algorithms were assumed to work in the extremely high signal to noise ratio (SNR) region. With the consideration of noise, noise reduction should be executed before phase reconstruction in wireless tomography, which means the de-noised amplitude-only total fields can be obtained from the large-scale received noise-polluted data. Otherwise, the poor performance of phase reconstruction in the presence of noise will cause severe imaging degradation.

Even with the aid of noise reduction, phase reconstruction cannot be well achieved [13]. Hence, the standard phase reconstruction [7], [14], [15] should be redesigned based on optimization theory. More relevant and accurate information about scattered fields should be incorporated into the standard phase reconstruction [7], [14], [15]. This paper proposes a hybrid system for wireless tomography. Some sensors in the system are advanced and sophisticated. These sensors can directly provide accurate full-data (both amplitude and phase information) total fields even in the presence of noise. Other sensors are cheap and simple, e.g., wireless communication devices. Those simple sensors, on the other hand, can only get the noise-polluted amplitude-only data for the received total fields. Kernel principal component analysis (PCA), which is a non-linear dimensionality reduction, will be used to perform noise reduction. Then, the accurate full-data total fields obtained by the advanced sensors will be exploited for phase reconstruction to retrieve the full-data scattered fields for those simple sensors. In this paper, we assume all incident fields are perfectly known. Single-frequency wireless tomography is considered. Meanwhile, the perfect wireless communication links are provided.

The key steps of novel wireless tomography in noisy environments can be summarized as follows:

1) **Noise reduction using kernel PCA;**

2) **Modified standard phase reconstruction for the simple sensors using the information from the advanced sensors;**

3) **Imaging.**

The system diagram of wireless tomography is shown in Fig. 1 compared with that of the self-coherent tomography shown in
The rest of the paper is organized as follows. In Section II the hybrid system for wireless tomography is presented. Section III, Section IV, and Section V will describe the theories for noise reduction, modified standard phase reconstruction, and imaging, respectively. Performance of wireless tomography will be provided in Section VI, followed by some remarks given in Section VII.

**II. HYBRID SYSTEM**

The hybrid system for wireless tomography is shown in Fig. 3. Some sensors in the system are advanced and sophisticated, while other sensors are low-cost and simple. Due to the energy limitation of the simple sensor, the data obtained by the simple sensor will be sent to the nearest advanced sensor. In this way, the lifetime of the simple sensor can be increased. Then the data obtained by the advanced sensors and received from the simple sensors will be transferred to the central controller of the hybrid system to perform signal processing for wireless tomography.

Assume the imaging area $\Omega$ is a circle plane with radius of $a$ which encloses the target. The measurement domain $\Gamma$ is a circle line with radius of $b$. There are total $N + L$ sensors on the circle line, in which $N$ sensors are simple with angles of $\theta_n$, $n = 1, 2, ..., N$ and $L$ sensors are advanced with angles of $\theta_l$, $l = 1, 2, ..., L$. How to determine the ratio of the simple sensors to the advanced sensors as well as the optimal deployment is out of scope of this paper. The source domain is also a circle line with radius of $c$. There are $I$ total sources with angles of $\theta_i$, $i = 1, 2, ..., I$. $\Omega$, $\Gamma$, and the source domain are concentric.

Horn antennas are used for all the sources and sensors, which keeps the excitation as close as possible to transverse magnetic (TM) polarization [16]. Two-dimensional near field diffraction tomography is considered. The background medium is assumed to be homogeneous with dielectric permittivity of $\varepsilon_b$ and magnetic permittivity of $\mu_0$. When the $i^{th}$ source sounds the imaging area using a single-tone signal with frequency of $\omega$, the scalar and nonlinear state equation can be written as [7], [14], [15],

$$E_{\text{tot},i}(\mathbf{r}) = E_{\text{inc},i}(\mathbf{r}) + k^2 \int_{\Omega} G(\mathbf{r} - \mathbf{r'})\chi(\mathbf{r'})E_{\text{tot},i}(\mathbf{r'})d\mathbf{r'}, \quad \mathbf{r} \in \Omega \quad (1)$$

where

- $k$ is the wavenumber given as,

$$k = \omega\sqrt{\mu_0\varepsilon_b}; \quad (2)$$

- $G(\mathbf{r} - \mathbf{r'})$ is the two-dimensional free space Green’s function given as [17], [18],

$$G(\mathbf{r} - \mathbf{r'}) = -\frac{\sqrt{-1}}{4}H_0^{(2)}(k|\mathbf{r} - \mathbf{r'}|) \quad (3)$$

in which $H_0^{(2)}$ is the Hankel function of the zero order and the second kind;

- $\chi(\mathbf{r})$ is the contrast function given as,

$$\chi(\mathbf{r}) = \varepsilon_\mathbf{r}(\mathbf{r}) - 1; \quad (4)$$

- target’s dielectric permittivity is $\varepsilon_\mathbf{r}(\mathbf{r})\varepsilon_b;$
• the conductivity of the target is not considered in this paper;
• \( E_{\text{inc},i}(\mathbf{r}) \) and \( E_{\text{tot},i}(\mathbf{r}) \) are incident and total fields evaluated in \( \Omega \) respectively.

Based on the state equation, the corresponding measurement equation for the \( l \)th advanced sensor is,

\[
E_{\text{tot},i}(\theta_l) = E_{\text{inc},i}(\theta_l) + E_{d,i}(\theta_l) + k^2 \int_{\Omega} G(\mathbf{r} - \mathbf{r}') \chi(\mathbf{r}') E_{\text{tot},i}(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r} = (b \cos(\theta_l), b \sin(\theta_l)) \in \Gamma
\]

where \( E_{\text{inc},i}(\theta_l) \), \( E_{d,i}(\theta_l) \), and \( E_{\text{tot},i}(\theta_l) \) are the direct incident field, scattered field, and total field measured by the \( l \)th advanced sensor respectively.

However, those simple sensors can only get noise-polluted amplitude-only data for the received total fields. Thus, the measurement equation for the \( n \)th simple sensor is,

\[
x_i(\theta_n) = |E_{\text{tot},i}(\theta_n) + n_i(\theta_n)|
\]

where \( E_{\text{tot},i}(\theta_n) \) is the true total field and \( n_i(\theta_n) \) is the complex-valued background noise the real and imaginary parts of which both follow Gaussian distribution with zero mean and the same variance. Though \( n_i(\theta_n) \) is the well known Gaussian noise, due to the non-linear operation in Eq. (6), the noise effect will be more severe than that caused by additive white Gaussian noise (AWGN).

Thus, the goal of noise reduction and phase reconstruction is to retrieve the accurate full-data scattered fields for those simple sensors, i.e., \( E_{d,i}(\theta_n) \), \( n = 1, 2, \ldots, N \), when the \( i \)th source sounds the target. This procedure will be repeated for each of the \( I \) sources.

III. NOISE REDUCTION

The task of noise reduction is to obtain the de-noised data \( \hat{E}_{\text{tot},i}(\theta_n) \) from the noise-polluted data \( x_i(\theta_n) \). When the \( i \)th source sounds the target, the vector representation of Eq. (6) can be simplified as,

\[
x = |\mathbf{E}_{\text{tot}} + \mathbf{n}|
\]

where

\[
x = \begin{bmatrix} x_i(\theta_1) \\ x_i(\theta_2) \\ \vdots \\ x_i(\theta_N) \end{bmatrix},
\]

\[
\mathbf{E}_{\text{tot}} = \begin{bmatrix} E_{\text{tot},i}(\theta_1) \\ E_{\text{tot},i}(\theta_2) \\ \vdots \\ E_{\text{tot},i}(\theta_N) \end{bmatrix}.
\]
and

\[ n = \begin{bmatrix} n_i(\theta_1) \\ n_i(\theta_2) \\ \vdots \\ n_i(\theta_N) \end{bmatrix} \]  \quad (10)

In the paper, \( |\tilde{E}_{\text{tot},i}(\theta_n)| \) is related to the de-noised data corresponding to the true data \( |E_{\text{tot},i}(\theta_n)|, n = 1, 2, \ldots, N \).

\( x \) constructs one sample vector in the dataset for noise reduction. The whole dataset can be achieved by executing a large number of measurements,

\[ x_m = |E_{\text{tot}} + n_m|, m = 1, 2, \ldots, M \]  \quad (11)

where \( M \) is the number of measurements (sample vectors) and \( n_m, m = 1, 2, \ldots, M \) for different measurements are assumed to be independent.

Hence, a set of sample vectors \( x_m, m = 1, 2, \ldots, M \) corresponding to the noise-polluted amplitude-only total fields for the simple sensors are obtained. If the full-data scattered fields are directly reconstructed from the noise-polluted amplitude-only total fields, the reconstructed values will be far from the true values. In order to solve this critical problem in wireless tomography, this paper proposes to perform noise reduction before phase reconstruction. Based on the observation that signal always lies in a low-dimensional space of the high-dimensional data, machine learning, especially dimensionality reduction, will be used to execute noise reduction.

Dimensionality reduction [19]–[23] tries to find a low-dimensional embedding of the high-dimensional data. PCA [24] is the best-known linear dimensionality reduction method. PCA works well for the high-dimensional data with linear relationship, but always fails in a non-linear situation. PCA can be extended to the non-linear counterpart by using a kernel [25]–[28], called kernel PCA [29].

Assume the original dataset contains \( M \) sample vectors, i.e., \( x_m \in \mathbb{R}^N, m = 1, 2, \ldots, M \). The corresponding sample vector of \( x_m \) after dimensionality reduction is \( y_m \in \mathbb{R}^K, m = 1, 2, \ldots, M \) where \( K << N \).

Kernel PCA uses the kernel function \( k(x_m, x_m') = \varphi(x_m) \cdot \varphi(x_m') \) to implicitly map the original data into a feature space \( F \), where \( \varphi \) is the mapping function and \( \cdot \) represents inner product. In \( F \), PCA can be applied.

If \( k \) is a valid kernel function, the kernel matrix

\[ K = \begin{bmatrix} k(x_1, x_1) & k(x_1, x_2) & \cdots & k(x_1, x_M) \\ k(x_2, x_1) & k(x_2, x_2) & \cdots & k(x_2, x_M) \\ \vdots & \vdots & \ddots & \vdots \\ k(x_M, x_1) & k(x_M, x_2) & \cdots & k(x_M, x_M) \end{bmatrix} \]  \quad (12)

must be a positive semi-definite matrix [30], [31].
Assume the mean of \( \varphi(x_m) \), \( m = 1, 2, \ldots, M \) is zero, i.e.,

\[
\frac{1}{M} \sum_{m=1}^{M} \varphi(x_m) = 0.
\]  
(13)

The covariance matrix of \( \varphi(x_m) \), \( m = 1, 2, \ldots, M \) is,

\[
C_F = \frac{1}{M} \sum_{m=1}^{M} \varphi(x_m) \varphi(x_m)^T.
\]  
(14)

In order to apply PCA in \( \mathbf{F} \), the eigenvectors \( \mathbf{v}_m^F \) of \( C_F \) are needed. The eigenvectors \( \mathbf{v}_m^F \) of \( C_F \) lie in the span of \( \varphi(x_j) \), \( j = 1, 2, \ldots, M \), i.e. [29],

\[
\mathbf{v}_m^F = \sum_{j=1}^{M} \alpha_{jm} \varphi(x_j).
\]  
(15)

It has been proven that \( \alpha_1, \alpha_2, \ldots, \alpha_M \) are eigenvectors of kernel matrix \( \mathbf{K} \) [29] and \( \alpha_{jm} \) are componentwise elements of \( \alpha_m \), i.e.,

\[
\alpha_m = \begin{bmatrix}
\alpha_{1m} \\
\alpha_{2m} \\
\vdots \\
\alpha_{Mm}
\end{bmatrix}.
\]  
(16)

The procedure of kernel PCA can be summarized into the following six steps:

1) Choose a kernel function \( k \);
2) Compute kernel matrix \( \mathbf{K} \);
3) Obtain the eigenvalues \( \lambda_1^K \geq \lambda_2^K \geq \cdots \geq \lambda_M^K \) and the corresponding eigenvectors \( \alpha_1, \alpha_2, \ldots, \alpha_M \) by diagonalizing \( \mathbf{K} \);
4) Normalize \( \mathbf{v}_k^F \) by [29]

\[
\alpha_k = \frac{\mathbf{v}_k^F}{\sqrt{\lambda_k^F}};
\]  
(17)

5) Constitute the basis of a subspace in \( \mathbf{F} \) from the normalized eigenvectors \( \mathbf{v}_k^F \), \( k = 1, 2, \ldots, K \);
6) Compute the projection of \( x_m \) on \( \mathbf{v}_k^F \), \( k = 1, 2, \ldots, K \) by

\[
y_{km} = \mathbf{v}_k^F \cdot \varphi(x_m) = \sum_{j=1}^{M} \alpha_{jk} k(x_j, x_m).
\]  
(18)

Hence, \( y_m \) corresponding to \( x_m \) can be written as,

\[
y_m = \begin{bmatrix}
y_{1m} \\
y_{2m} \\
\vdots \\
y_{Km}
\end{bmatrix}.
\]  
(19)

So far the mean of \( \varphi(x_m) \), \( m = 1, 2, \ldots, M \) has been assumed to be zero. In fact, the data with zero mean in the feature
space are,
\[ \varphi(x_m) - \frac{1}{M} \sum_{m=1}^{M} \varphi(x_m). \] (20)

The kernel matrix for the centering data can be derived as [29],
\[ \tilde{K} = HKH \] (21)
in which \( H \) is the so-called centering matrix defined as,
\[ H = I - \frac{1}{M} \mathbf{1} \mathbf{1}^T \] (22)
where \( I \) is an identity matrix; \( \mathbf{1} = (1, 1, \cdots, 1)^T \) is a \( M \times 1 \) vector; and \( T \) represents transpose operator.

In order to apply kernel PCA for noise reduction, the pre-image \( \tilde{x}_m \) (in the original space) of \( y_m \) (in the feature space) will be calculated if \( x_m \) is to be de-noised. The distance-constraint based method used here for noise reduction explores the distance relationship between the original space and the feature space for some specific kernels [32]. It tries to find the distance between \( \tilde{x}_m \) and \( x_j \), \( j = 1, 2, ..., M \) once the distance between \( y_m \) and \( \varphi(x_j) \) is known. \( d(x_{m1}, x_{m2}) \) is used to represent distance between two vectors \( x_{m1} \) and \( x_{m2} \).

The squared distance between \( y_m \) and \( \varphi(x_j) \) can be derived as [33],
\[ d^2(y_m, \varphi(x_j)) \]
\[ = (k_{x_m} + \frac{1}{M} K_1 - 2k_{x_j})^T H^T M (k_{x_m} - \frac{1}{M} K_1) \]
\[ + \frac{1}{M^2} \mathbf{1}^T k_1 + K_{jj} - \frac{2}{M} \mathbf{1}^T k_{x_j} \] (23)
where
\[ k_{x_m} = \begin{bmatrix} k(x_m, x_1) \\ k(x_m, x_2) \\ \vdots \\ k(x_m, x_M) \end{bmatrix}, \] (24)
\[ K_{jj} = k(x_j, x_j), \] (25)
and
\[ M = \sum_{k=1}^{K} \frac{1}{\lambda_k} \hat{\alpha}_k \hat{\alpha}_k^T \] (26)
in which \( \lambda_k \) and \( \hat{\alpha}_k \) are the \( k^{th} \) largest eigenvalue and corresponding column eigenvector of \( \tilde{K} \), respectively.

By making use of the distance relationship between the original space and the feature space [32], \( d^2(\tilde{x}_m, x_j), j = 1, 2, ..., M \) can be obtained. In the paper, the radial basis kernel defined in Eq. (27) with parameter \( \gamma = 2 \) will be used
\[ k(x_{m1}, x_{m2}) = \exp(-\gamma \| x_{m1} - x_{m2} \|^2). \] (27)
Then \( d^2(\tilde{x}_m, x_j), j = 1, 2, ..., M \) will be expressed as [33],
\[
d^2(\tilde{x}_m, x_j) = -\frac{1}{\gamma} \log \left( \frac{1}{2} \left( k(\tilde{x}_m, \tilde{x}_m) + k(x_j, x_j) - d^2(y_m, \varphi(x_j)) \right) \right)
\]
(28)

These distances (in the original space) are exploited to constrain the embedding of the pre-image \( \tilde{x}_m \) [33], [34]. Only the local neighborhood structure needs to be preserved, so twenty nearest neighbors are selected to determine the location of \( \tilde{x}_m \) [33].

Assume these twenty neighbors are \( x_{j_1}, x_{j_2}, ..., x_{j_{20}} \). Define
\[
X = [x_{j_1}, x_{j_2} \cdots x_{j_{20}}]
\]
(29)
and perform singular value decomposition (SVD) to \( XH \) as [33],
\[
XH = U\Lambda V^T
\]
(30)
\[
= UZ.
\]
(31)
where \( H \) is the centering matrix defined in Eq. (22) with size of \( 20 \times 20 \) and \( Z = [z_1 z_2 \cdots z_{20}] \).

Define
\[
d_0^2 = \begin{bmatrix}
\|z_1\|^2_2 \\
\|z_2\|^2_2 \\
\vdots \\
\|z_{20}\|^2_2
\end{bmatrix}
\]
(32)
and
\[
d^2 = \begin{bmatrix}
d^2(\tilde{x}_m, x_{j_1}) \\
d^2(\tilde{x}_m, x_{j_2}) \\
\vdots \\
d^2(\tilde{x}_m, x_{j_{20}})
\end{bmatrix}
\]
(33)

Hence, \( \tilde{x}_m \) can be calculated as [33],
\[
\tilde{x}_m = -\frac{1}{2} U\Lambda^{-1} V^T (d^2 - d_0^2) + \frac{1}{20} \sum_{i=1}^{20} x_{j_i}
\]
(34)

Finally, the de-noised data \( |\tilde{E}_{tot}| = \tilde{x}_m \) can be achieved with the componentwise element \( |\tilde{E}_{tot,i}(\theta_n)| \).

IV. Modified Standard Phase Reconstruction

In diffraction tomography or inverse scattering, phase reconstruction means reconstructing the full-data scattered fields from the full-data incident fields and the amplitude-only total fields [7], [14], [15]. Mathematically speaking, phase reconstruction tries to reconstruct \( E_{d,i}(\theta_n) \) from \( E_{inc,i}(\theta_n) \) and \( |E_{tot,i}(\theta_n)|^2 \).

In order to solve the phase reconstruction problem, a sequence of non-linear equations will be built. However if the number of variables to be solved is similar to or larger than the number of equations, the problem will be underdetermined, which will
make the solution non-unique and sensitive to disturbance. Thus, we should find the key variables, which are called degrees of freedom, inside the phase reconstruction problem to reduce the number of variables to be solved.

The operator related to the scattered field is defined as [14],

\[
B[E_{d,i}(\theta_n)] = |E_{\text{tot},i}(\theta_n)|^2 - |E_{\text{inc},i}(\theta_n)|^2
\]

\[
= (E_{d,i}(\theta_n) + E_{\text{inc},i}(\theta_n))(E_{d,i}(\theta_n) + E_{\text{inc},i}(\theta_n))^* - |E_{\text{inc},i}(\theta_n)|^2
\]

\[
= |E_{d,i}(\theta_n)|^2 + (E_{d,i}(\theta_n))(E_{\text{inc},i}(\theta_n))^* + (E_{\text{inc},i}(\theta_n))(E_{d,i}(\theta_n))^*
\]

\[
= |E_{d,i}(\theta_n)|^2 + 2\text{Re}[E_{d,i}(\theta_n)E_{\text{inc},i}(\theta_n)^*]
\]

in which \( * \) stands for conjugate operator.

It has been thoroughly investigated [35] that the scattered field can be accurately represented with \( 2ka \) Fourier harmonics which have the finite degrees of freedom. Thus,

\[
E_{d,i}(\theta_n) = \sum_{q=-ka}^{ka} c_q e^{jq\theta_n}
\]

\[
= \sum_{q=-ka}^{ka} (x_q + jy_q)e^{jq\theta_n}, \quad n = 1, 2, \ldots, N
\]

(36)

in which \( c_q = x_q + jy_q \). The Fourier harmonic coefficients \( x_q \) and \( y_q \) are the actual variables to be retrieved. \( N \) non-linear equations will be established as,

\[
|E_{\text{tot},i}(\theta_n)|^2 - |E_{\text{inc},i}(\theta_n)|^2
\]

\[
= \left| \sum_{q=-ka}^{ka} (x_q + jy_q)e^{jq\theta_n} \right|^2 + 2\text{Re}\left( \sum_{q=-ka}^{ka} (x_q + jy_q)e^{jq\theta_n}E_{\text{inc},i}(\theta_n)^* \right), \quad n = 1, 2, \ldots, N
\]

(37)

with the known \( E_{\text{inc},i}(\theta_n) \) and \( |E_{\text{tot},i}(\theta_n)|^2 \).

In order to make the problem overdetermined, the number of variables which is \( 4ka + 2 \) should be less than \( N \). As long as the optimal \( x_q \) and \( y_q \) are obtained, \( E_{d,i}(\theta_n) \) can be calculated by Eq. (36).

In the paper, the standard phase reconstruction means retrieving the full-data scattered fields directly from the noise-polluted amplitude-only total fields defined in Eq. (6) for those simple sensors.

However, in wireless tomography, even after noise reduction, \( |E_{\text{tot},i}(\theta_n)| \) cannot be exactly obtained. \( \tilde{E}_{\text{tot},i}(\theta_n) \) will be used instead of \( |E_{\text{tot},i}(\theta_n)| \). The slight deviation of \( \tilde{E}_{\text{tot},i}(\theta_n) \) from \( |E_{\text{tot},i}(\theta_n)| \) will cause the reconstructed scattered fields to be far from the true scattered fields.

Thus, we should regularize \( x_q \) and \( y_q \) to force the reconstructed scattered fields to approach the true scattered fields. In order to achieve this goal, the accurate full-data scattered fields provided by the advanced sensors will be explored to set up \( L \) linear equations,

\[
E_{\text{tot},i}(\theta_l) - E_{\text{inc},i}(\theta_l) = \sum_{q=-ka}^{ka} (x_q + jy_q)e^{jq\theta_l}, \quad l = 1, 2, \ldots, L.
\]

(38)
Finally, the MATLAB function fsolve will be adopted to give the solution to the established hybrid equation set with \( N \) non-linear equations defined in Eq. (37) and \( L \) linear equations defined in Eq. (38).

V. IMAGING

Born iterative method [36] will be exploited to perform coherent tomography imaging. Imaging of the target with the close-in sensors [37]–[39] shown in Fig. 3 will be taken into account.

All the sources take turns to sound the target. The advanced sensors can measure the accurate full-data scattered fields directly, while the scattered fields from the simple sensors will be reconstructed by noise reduction and modified standard phase reconstruction mentioned before. After all the sources sound the target, the multistatic data matrix \( E \) corresponding to scattered fields can be obtained.

Generally, assume the imaging area is discretized into \( S \) cells with the same area of \( \Delta \). The location of each cell is \( \mathbf{r}_s, s = 1, 2, \ldots, S \) and the contrast of each cell is \( \chi_s, s = 1, 2, \ldots, S \). The sensor location is \( \mathbf{\beta}_j, j = 1, 2, \ldots, N + L \). The source location is \( \mathbf{\gamma}_i, i = 1, 2, \ldots, I \). In this paper,

\[
\mathbf{\beta}_j = (b \cos(\theta_j), b \sin(\theta_j))
\]

and

\[
\mathbf{\gamma}_i = (c \cos(\theta_i), c \sin(\theta_i)).
\]

Based on the measurement equation shown in Eq. (5), \( E_{ji} \) is expressed as,

\[
E_{ji} = \sum_{s=1}^{S} \chi_s G (\mathbf{\beta}_j - \mathbf{r}_s) E_{\text{tot},i}(\mathbf{r}_s)(k^2 \Delta)
\]

where \( E_{ji} \) represents the scattered field measured by the \( j^{th} \) sensor when the \( i^{th} \) source sounds the target and the total field \( E_{\text{tot},i}(\mathbf{r}_s) \) can be calculated from the state equation shown in Eq. (1) when the \( i^{th} \) source sounds the target.

Define \( \mathbf{e} \in \mathbb{C}^{I(N+L) \times 1} \) as \( \mathbf{e}_{(j+(i-1)(N+L))1} = E_{ji} \), i.e.,

\[
\mathbf{e} = \begin{bmatrix}
E_{11} \\
E_{21} \\
\vdots \\
E_{(N+L)1} \\
E_{12} \\
\vdots \\
E_{(N+L)L}
\end{bmatrix}
\]
Define $\chi$ as $\chi_{s1} = \chi_s$, i.e.,

$$\chi = \begin{bmatrix}
\chi_1 \\
\chi_2 \\
\vdots \\
\chi_S
\end{bmatrix}. \tag{43}$$

Define $B \in C^{(N+L) \times S}$ as $B_{(j+(i-1)(N+L))s} = G(\beta_j - \mathbf{r}_s)E_{tot,i}(\mathbf{r}_s)(k^2\Delta)$, i.e.,

$$B = \begin{bmatrix}
G(\beta_1 - \mathbf{r}_1)E_{tot,1}(\mathbf{r}_1)(k^2\Delta) & \cdots & G(\beta_1 - \mathbf{r}_S)E_{tot,1}(\mathbf{r}_S)(k^2\Delta) \\
G(\beta_2 - \mathbf{r}_1)E_{tot,1}(\mathbf{r}_1)(k^2\Delta) & \cdots & G(\beta_2 - \mathbf{r}_S)E_{tot,1}(\mathbf{r}_S)(k^2\Delta) \\
\vdots & \ddots & \vdots \\
G(\beta_{(N+L)} - \mathbf{r}_1)E_{tot,1}(\mathbf{r}_1)(k^2\Delta) & \cdots & G(\beta_{(N+L)} - \mathbf{r}_S)E_{tot,1}(\mathbf{r}_S)(k^2\Delta)
\end{bmatrix}. \tag{44}$$

Define $U \in C^{S \times I}$ as $U_{si} = E_{tot,i}(\mathbf{r}_s)$, i.e.,

$$U = \begin{bmatrix}
E_{tot,1}(\mathbf{r}_1) & E_{tot,2}(\mathbf{r}_1) & \cdots & E_{tot,I}(\mathbf{r}_1) \\
E_{tot,1}(\mathbf{r}_2) & E_{tot,2}(\mathbf{r}_2) & \cdots & E_{tot,I}(\mathbf{r}_2) \\
\vdots & \ddots & \vdots & \vdots \\
E_{tot,1}(\mathbf{r}_S) & E_{tot,2}(\mathbf{r}_S) & \cdots & E_{tot,I}(\mathbf{r}_S)
\end{bmatrix}. \tag{45}$$

Define $G \in C^{S \times I}$ as $G_{si} = G(\mathbf{r}_s - \gamma_i)$, i.e.,

$$G = \begin{bmatrix}
G(\mathbf{r}_1 - \gamma_1) & G(\mathbf{r}_1 - \gamma_2) & \cdots & G(\mathbf{r}_1 - \gamma_I) \\
G(\mathbf{r}_2 - \gamma_1) & G(\mathbf{r}_2 - \gamma_2) & \cdots & G(\mathbf{r}_2 - \gamma_I) \\
\vdots & \ddots & \ddots & \vdots \\
G(\mathbf{r}_S - \gamma_1) & G(\mathbf{r}_S - \gamma_2) & \cdots & G(\mathbf{r}_S - \gamma_I)
\end{bmatrix}. \tag{46}$$

Define $A \in C^{S \times S}$ as,

$$A = \begin{bmatrix}
1 & -\chi_2G(\mathbf{r}_1 - \mathbf{r}_2)(k^2\Delta) & \cdots & -\chi_SG(\mathbf{r}_1 - \mathbf{r}_S)(k^2\Delta) \\
-\chi_1G(\mathbf{r}_2 - \mathbf{r}_1)(k^2\Delta) & 1 & \cdots & -\chi_SG(\mathbf{r}_2 - \mathbf{r}_S)(k^2\Delta) \\
\vdots & \ddots & \ddots & \vdots \\
-\chi_1G(\mathbf{r}_S - \mathbf{r}_1)(k^2\Delta) & -\chi_2G(\mathbf{r}_S - \mathbf{r}_2)(k^2\Delta) & \cdots & 1
\end{bmatrix}. \tag{47}$$

Based on the state equation shown in Eq. (1)

$$AU = G \tag{48}$$

Born iterative method is used to get $\chi$ by the following steps [9], [17], [36], [40]–[42]:

1. Initialize $\chi_0 = 0$.
2. For $k = 0, 1, 2, \ldots$
   - Compute $\chi_k$ from $AU = G$.
   - Update $\chi_k$ using an iterative scheme.
3. Stop when $\chi_k$ converges.
1) Set $\chi^{(0)}$ to be zero; $t = -1$;

2) $t = t + 1$; get $B^{(t)}$ based on Eq. (44), Eq. (45), Eq. (46), Eq. (47), and Eq. (48) using $\chi^{(t)}$;

3) Solve the inverse problem in Eq. (49) by Tikhonov regularization to get $\chi^{(t+1)}$

$$e = B^{(t)}\chi^{(t+1)};$$ (49)

4) If $\chi$ converges, Born iterative method is stopped; otherwise Born iterative method goes to step 2.

VI. PERFORMANCE

Experimental data provided by the Institute Fresnel in Marseille, France are used to demonstrate the concept of wireless tomography and verify the corresponding algorithms. The free space experimental setup and measurement precision of data are mentioned in [16]. These experimental data have been widely used to test the novel algorithms for inverse scattering and RF tomography [43]–[47].

First, the data corresponding to the hybrid target called FoamDieInt are exploited. TM polarization is measured. This hybrid target shown in Fig. 4 consists of a foam cylinder (SAITEC SBF 300) with diameter of 80 mm and relative permittivity of $1.45 \pm 0.15$ as well as a plastic cylinder (berylon) with diameter of 31 mm and relative permittivity of $3 \pm 0.3$ [16]. The frequency of the sounding signal is 2 GHz. The imaging area, i.e., the investigated domain, is 150 mm by 150 mm. The imaging resolution is 5 mm. The number of sources is 8 and the number of total sensors is 240. The distance between each source and the center of the imaging area is 1.67 m. The distance between the center of the imaging area and each sensor is also 1.67 m. The source positions are taken with a step of $45^\circ$ from $0^\circ$ [16]. The sensor positions are taken with a step of $1^\circ$ from $60^\circ$ [16]. If all the sensors are advanced, the imaging result using accurate full-data scattered fields is shown in Fig. 5.

For verifying the concept of wireless tomography in the presence of noise, the man-made noises are added into the experimental data. The number of measurements $M$ is 200. SNR is equal to $15 dB$. The number of advanced sensors for each source is 2. These advanced sensors are uniformly deployed among all the sensors. The reconstructed amplitude information and phase information using the proposed phase reconstruction including noise reduction and modified standard phase reconstruction for the simple sensors are shown in Fig. 6 and Fig. 7 respectively when the first source sounds the target. The reconstructed full-data scattered fields for the simple sensors by the proposed phase reconstruction approach the true scattered fields. The imaging result of wireless tomography is shown in Fig. 8. If Born iterative method is applied to the accurate full-data scattered fields measured by two advanced sensors for each source, the imaging result is shown in Fig. 9. If self-coherent tomography is applied to the noise-polluted amplitude-only total fields measured by simple sensors, Born iterative method cannot converge to get the imaging result. Hence, with the aid of a small amount of advanced sensors, the imaging performance of wireless tomography in noisy environments is acceptable and promising. Wireless tomography explores a priori information from advanced sensors and the state of the art mathematics, e.g., machine learning and optimization theory, to extract knowledge from the large-scale noisy data provided by simple sensors. Wireless tomography can be treated as one realization of data-enabled science and engineering for “Big Data” in remote sensing.

The number of advanced sensors for each source is increased to 3. These advanced sensors are also uniformly deployed
among all the sensors. The reconstructed amplitude information and phase information using the proposed phase reconstruction are shown in Fig. 10 and Fig. 11 respectively when the first source sounds the target. The imaging result of wireless tomography is shown in Fig. 12. If Born iterative method is applied to the accurate full-data scattered fields measured from three advanced sensors for each source, the imaging result is shown in Fig. 13. If SNR is increased to $20dB$, the imaging result of wireless tomography is shown in Fig. 14.

Then, the data corresponding to the hybrid target called FoamDielExt are considered. TM polarization is also measured. This hybrid target consists of a foam cylinder (SAITEC SBF 300) with diameter of 80 mm and relative permittivity of $1.45 \pm 0.15$ as well as a plastic cylinder (berylon) with diameter of 31 mm and relative permittivity of $3 \pm 0.3$ [16]. The experiment setup for FoamDielExt is the same as the experiment setup for FoamDielInt. The number of advanced sensors for each source is 3. SNR is equal to $20dB$. The imaging result of wireless tomography is shown in Fig. 15. If Born iterative method is applied to the accurate full-data scattered fields measured from three advanced sensors for each source, the imaging result is shown in Fig. 16.

Finally, the data corresponding to the hybrid target called FoamTwinDiel are used. TM polarization is also measured. This hybrid target consists of a foam cylinder (SAITEC SBF 300) with diameter of 80 mm and relative permittivity of $1.45 \pm 0.15$ as well as two plastic cylinders (berylon) with diameter of 31 mm and relative permittivity of $3 \pm 0.3$ [16]. The number of sources is 18. The source positions are taken with a step of $20^\circ$ from $0^\circ$ [16]. The number of advanced sensors for each source is 3. SNR is equal to $20dB$. The imaging result of wireless tomography is shown in Fig. 17. If Born iterative method is applied to the accurate full-data scattered fields measured from three advanced sensors for each source, the imaging result is shown in Fig. 18. In sum, wireless tomography gives more accurate imaging results. Target characteristics and locations can be easily identified. Due to the limited number of advanced sensors for each source, only using the accurate full-data scattered fields from advanced sensors cannot provide full information to image the target.

VII. CONCLUSION

This paper has followed the novel concept of wireless tomography and dealt with wireless tomography in noisy environments for potential real-world applications. We would like to explore wireless tomography using currently available commercial wireless communication network or (the next generation) cognitive radio network. The bottleneck is to retrieve accurate phase information from noisy measurements when wireless communication devices are used. This paper has addressed this fundamental issue. The experimental data have been used to demonstrate the concept of wireless tomography and verify the corresponding algorithms. From the simulation results, the imaging performance of wireless tomography is very promising even in the presence of noise.

Eventually, we would like to take wireless tomography from the lab to the real-world situations, e.g., battle field, search and rescue, surveillance, reconnaissance, and so on. For the large-scale deployment of sensors for wireless tomography, smart phone, universal software radio peripheral (USRP), or wireless open access research platform (WARP) can be used as simple sensors to perform sensing function and data acquisition. We will continue this line of work and set up our own hybrid system to demonstrate the real-world wireless tomography [48].
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Fig. 1. The system diagram for wireless tomography.

Fig. 2. The system diagram for self-coherent tomography.
Fig. 3. The hybrid system for wireless tomography.

Fig. 4. The hybrid target called FoamDieInt in French data [16].
Fig. 5. The imaging result for FoamDieIInt using accurate full-data scattered fields.

Fig. 6. The reconstructed amplitude information when the number of the advanced sensors for each source is 2 and SNR is $15dB$. (Proposed phase reconstruction includes noise reduction and modified standard phase reconstruction for the simple sensors using the information from the advanced sensors.)

Fig. 7. The reconstructed phase information when the number of the advanced sensors for each source is 2 and SNR is $15dB$. (Proposed phase reconstruction includes noise reduction and modified standard phase reconstruction for the simple sensors using the information from the advanced sensors.)
Fig. 8. The imaging result of wireless tomography for FoamDieInt when the number of the advanced sensors for each source is 2 and SNR is 15dB.

Fig. 9. The imaging result for FoamDieInt when Born iterative method is applied to the accurate full-data scattered fields from two advanced sensors for each source.

Fig. 10. The reconstructed amplitude information when the number of the advanced sensors for each source is 3 and SNR is 15dB. (Proposed phase reconstruction includes noise reduction and modified standard phase reconstruction for the simple sensors using the information from the advanced sensors.)
Fig. 11. The reconstructed phase information when the number of the advanced sensors for each source is 3 and SNR is 15dB. (Proposed phase reconstruction includes noise reduction and modified standard phase reconstruction for the simple sensors using the information from the advanced sensors.)

Fig. 12. The imaging result of wireless tomography for FoamDieIInt when the number of the advanced sensors for each source is 3 and SNR is 15dB.

Fig. 13. The imaging result for FoamDieIInt when Born iterative method is applied to the accurate full-data scattered fields from three advanced sensors for each source.
Fig. 14. The imaging result of wireless tomography for FoamDieInt when the number of the advanced sensors for each source is 3 and SNR is 20dB.

Fig. 15. The imaging result of wireless tomography for FoamDieExt when the number of the advanced sensors for each source is 3 and SNR is 20dB.

Fig. 16. The imaging result for FoamDieExt when Born iterative method is applied to the accurate full-data scattered fields from three advanced sensors for each source.
Fig. 17. The imaging result of wireless tomography for FoamTwinDiel when the number of the advanced sensors for each source is 3 and SNR is 20dB.

Fig. 18. The imaging result for FoamTwinDiel when Born iterative method is applied to the accurate full-data scattered fields from three advanced sensors for each source.