Spectrum Sensing by Cognitive Radios at Very Low SNR

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Introduction and Motivation

- FCC is developing rules for dynamic use of radio spectrum
  - Radio spectrum is currently under-utilized
  - Empty TV bands
- Cognitive Radios (IEEE 802.22)
  - Sense spectrum and identify **White Space** for dynamic use
  - Not allowed to interfere with licensed transmissions (TV & Wireless Mic.)
- Market Opportunities
  - Google, Microsoft, and Motorola (Wi-Fi)
  - Qualcomm (improve capacity of cellular networks)
  - Philips, Dell, et. al. (new wireless devices)
- Technical Challenges: **Spectrum Sensing**
  - Detect signals at very low SNR (-10 to -20 dB)
  - Quick sensing (less than 2 seconds)
  - Multipath fading and shadowing: hidden node problem
Outline

- Overview of Spectrum Sensing Techniques
- Spectral Feature Detection
  - An asymptotically optimal detector
  - Equivalent to the likelihood ratio test (LRT) at very low SNR
- Spectral Feature Selection
  - Optimization Analysis (non-convex problem)
- Threshold Estimation
  - Moment Method
- Simulation
  - Detects primary TV signals (NTSC and ATSC) at SNR around -20 dB
Spectrum Sensing Techniques

- Energy Detection (Radio-Meter)
  - Non-coherent detection (synchronization not needed)
  - Optimal if only noise power is known

- Matched Filtering
  - Coherent detection
  - Synchronization needed (if a deterministic sequence is known)
  - Performance better than energy detection

- Feature Detection
  - Non-coherent method (synchronization not needed)
  - Cyclostationarity (exploits inherent periodicities in first- and second-order statistics, assuming that the modulation scheme is known)
Spectrum Sensing at very low SNR

- Traditional detection techniques are no longer applicable
  - Energy detection (failure due to noise uncertainty)
  - Matched filtering (difficult to obtain synchronization)
  - Cyclo-stationary detector (too complex)

- Our strategy: Spectral Feature Detection
  - Exploits spectral features of primary signals

**ATSC** (Digital TV standard)
- Pilot tone at 309 KHz above band edge
- 8VSB Modulation

**NTSC** (Analog TV standard)
- Video carrier (AM), 1.25 MHz above band edge
- Color carrier (QAM), 3.58 MHz above video carrier
- Audio Carrier (FM), 4.5 MHz above video carrier
Spectral Feature Detector

- Extract the unique spectral features of a specific TV signal
- Use the \textit{a priori} known feature to match the signal under detection
- Spectrum sensing is modeled as a binary hypothesis test:
  \[ \mathcal{H}_0 : \quad y(l) = v(l) \]
  \[ \mathcal{H}_1 : \quad y(l) = x(l) + v(l) \quad v(l) \sim \mathcal{CN}(0, \sigma_v^2) \]

- PSD is estimated via \( n \)-point DFT (periodogram)
  \[ \mathcal{H}_0 : \quad S_Y^{(n)}(k) = \hat{\sigma}_v^2 \]
  \[ \mathcal{H}_1 : \quad S_Y^{(n)}(k) = S_X^{(n)}(k) + \hat{\sigma}_v^2 \]

- Decision rule
  \[ T_n = \frac{1}{n} \sum_{k=0}^{n-1} S_Y^{(n)}(k) S_X^{(n)}(k) \]
  \[ \begin{array}{c}
  \mathcal{H}_1 \\
  \mathcal{H}_0
  \end{array} \begin{array}{c}
  \geq \quad t_n
  \end{array} \]

  where \( S_X^{(n)}(k) \) is the pre-stored spectral feature (e.g., ATSC or NTSC).
Asymptotical Optimality

Assumption: TV signals \( x = [x(0), x(1), \ldots, x(n-1)]^T \) are a second-order stationary stochastic (Gaussian) process

\[
\mathcal{H}_0 : \quad y \sim \mathcal{CN} \left(0, \sigma_v^2 \mathbf{I}\right)
\]

\[
\mathcal{H}_1 : \quad y \sim \mathcal{CN} \left(0, \Sigma_n + \sigma_v^2 \mathbf{I}\right)
\]

where \( \Sigma_n = \mathbb{E}(xx^T) \) is the covariance matrix.

Optimal likelihood ratio test (LRT) at very low SNR is given by

\[
T_{\text{LRT},n} = \frac{1}{n} y^T \Sigma_n y \quad \mathcal{H}_1 \quad \geq \quad t_{\text{LRT}}
\]

\[
\mathcal{H}_0
\]

(Proof in backup slides)

\( T_n \) is asymptotically equivalent to \( T_{\text{LRT},n} \) for large block size \( n \), i.e.,

\[
\lim_{n \to \infty} |T_n - T_{\text{LRT},n}| = 0
\]

(Proof in backup slides)

The spectral feature detector is asymptotically optimal at very low SNR and for large block size \( n \).
What's a good spectral feature for detection? (useful insights for signal design)

Detector (mean) under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$

$$T_{n,0} = \frac{1}{n} \sigma_v^2 \sum_{k=0}^{n-1} S_X(k) = \sigma_v^2 P_x$$

Primary transmit power (constant)

$$T_{n,1} = \sigma_v^2 P_x + \frac{1}{n} \sum_{k=0}^{n-1} S_X^2(k)$$

The difference determines the detection performance

How to choose $\{S_X(k)\}$ to maximize the difference?

maximize $T_{n,1} - T_{n,0}$

s.t. $\frac{1}{n} \sum_{k=0}^{n-1} S_X(k) = P_x$

$s_X(k) \geq 0, \quad k = 0, 1, \ldots, n - 1$
What Feature is Best (or Worst)?

- Maximizes a convex function over a hyper-plane (non-convex)

\[
\begin{align*}
\max & \sum_{k=0}^{n-1} S_X^2(k) \\
\text{s.t.} & \sum_{k=0}^{n-1} S_X(k) = nP_x \\
S_X(k) & \geq 0, \quad k = 0, 1, \ldots, n-1
\end{align*}
\]

- Optimal: all transmit power concentrated in a frequency bin (sinusoid)

\[
\begin{cases}
S_X(j) = nP_x, & j \in \{0,1,\ldots,n-1\} \\
S_X(k) = 0, & 0 \leq k \leq n-1, \text{ and } k \neq j
\end{cases}
\]

\[
T_{n,1} - T_{n,0} = n^2 P_x^2
\]

- Worst: transmit power evenly distributed (white Gaussian like)

\[
S_X(k) = P_x, \quad k = 0, 1, \ldots, n-1
\]

\[
T_{n,1} - T_{n,0} = nP_x^2
\]

Sharp spectral feature provides better detection performance.
Threshold Estimation

- Suppose $\mathcal{H}_0$ is true (absence of primary signals)
- Periodogram is *chi-squared* distributed, i.e., $\hat{S}_Y(k) \sim \chi^2_2$
- The detector is a linear combination of chi-squared R.V.

$$T_n = \frac{1}{n} \sum_{k=0}^{n-1} \hat{S}_Y(k) S_X(k) \begin{cases} \mathcal{H}_1 & \text{if} \ \mathcal{H}_0 \leq \mathcal{H}_1 \\ \mathcal{H}_0 & \text{if} \ \mathcal{H}_0 \leq \mathcal{H}_0 \end{cases}$$

What is its distribution?

- Moment Method: calculate the logarithm of its moment generating function

$$g(\tau) = \log \mathbb{E}(e^{\tau T_L}) = \sum_{k=0}^{n-1} \log \mathbb{E}(e^{\tau \hat{S}_Y(k) S_X(k)/n}) = - \sum_{k=0}^{n-1} \log (1 - \tau \theta_k)$$

- Cumulants

$$c_n = g^{(n)}(0) = (n - 1)! \sum_{k=0}^{n-1} \theta_k^n$$

$$\theta_k = \frac{\sigma_v^2 S_X(k)}{n}$$
Chi-Square Approximation

- **High-order statistics**
  \[ c_1 \] \textit{Mean}
  \[ c_2 \] \textit{Variance}
  \[ S_T = \frac{c_3}{\sqrt{c_2^3}} \] \textit{Skewness} (measure of asymmetry)
  \[ K_T = \frac{c_4}{c_2^2} \] \textit{Kurtosis} (measure of peakedness)

- **Non-central chi-squared distribution** \( \chi^2_\eta(\delta) \)
  \[ S_{\chi^2} = \frac{2\sqrt{2}(\eta + 3\delta)}{(\eta + 2\delta)^{3/2}} \]
  \[ K_{\chi^2} = \frac{12(\eta + 4\delta)}{(\eta + 2\delta)^2} \]

- **Approximation (solved for \( \eta \) and \( \delta \))**
  \[
  \text{minimize} \quad \Delta_K = |K_T - K_{\chi^2}|
  \]
  \[
  \text{s.t.} \quad S_T = S_{\chi^2}
  \]
Prob. False Alarm

- Prob. false alarm is the tail prob.:
  \[
  Pr\left(T_L > t\right) = Pr\left(\frac{T_L - c_1}{\sqrt{c_2}} > t^*\right)
  \approx Pr\left(\frac{\chi^2_\eta(\delta) - \mu\chi^2}{\sigma\chi^2} > t^*\right)
  = Pr\left(\chi^2_\eta(\delta) > t^*\sigma\chi^2 + \mu\chi^2\right)
  \]
  \[
  t^* = \frac{t - c_1}{\sqrt{c_2}}
  \]
  \[
  \mu\chi^2 = \eta + \delta
  \]
  \[
  \sigma_\chi^2 = \sqrt{2(\eta + 2\delta)}
  \]

- Cumulative distribution function (CDF) of non-central chi-square:
  \[
  P(x; \eta, \delta) = e^{-\delta/2} \sum_{k=0}^{\infty} \frac{(\delta/2)^k}{k!} F(x; \eta + 2k)
  \]

- CDF of central chi-square:
  \[
  F(x; \eta) = \frac{\gamma(\eta/2, x/2)}{\Gamma(\eta/2)}
  \]

Incomplete gamma function
\[
\gamma(z, x) := \int_0^x \tau^{z-1} e^{-\tau} d\tau
\]

Gamma function
\[
\Gamma(z) := \int_0^{\infty} \tau^{z-1} e^{-\tau} d\tau
\]
Numerical Results

Threshold of spectral feature detector can be estimated numerically.
Computational Complexity

- Likelihood Ratio Test (LRT)
  - Computing covariance matrix (off-line): $O(n^2)$
  - Quadratic detector (on-line): $O(n^2)$

- Eigenvalue Based Spectrum Sensing (Zeng, etc.)
  - Computing covariance matrix (on-line): $O(n^2)$
  - Eigen-decomposition (on-line): $O(n^3)$

- Cyclo-stationary Detection
  - Computing the auto-correlation function (on-line): $O(n^2)$

- Spectral Feature Detector
  - Calculating periodogram (using FFT): $O(n \log_2 n)$
  - Correlation (on-line): $O(n)$

The spectral feature detector has less computational complexity.
Simulation (Digital TV - ATSC)

- Prob. false alarm less than 0.001

- Plot showing the relationship between SNR (dB) and the probability of missed detection for different signal-to-noise ratios.

- Legend:
  - AWGN (24 ms)
  - AWGN (48 ms)
  - ITU Ped-B (24 ms)
  - ITU Ped-B (48 ms)

- Small inset showing the power spectral density.
Simulation (Analog TV - NTSC)

- Prob. false alarm less than 0.001

![Graph showing probability of missed detection vs. SNR (dB)]

- AWGN (6 ms)
- AWGN (12 ms)
- ITU Ped-B (12 ms)
- ITU Ped-B (24 ms)
Summary

- Spectral feature detector is asymptotically optimal
  - At very low SNR
  - With a large block size

- Reliably detects TV signals at SNR levels around -20 dB or so.

- Non-coherent detection (synchronization is not needed)

- Low computational complexity compared with the LRT detector.

- Threshold can be estimated numerically.

- Feature selection: prefer sharp spectral features
Thank you!