TARGET DETECTION WITH FUNCTION OF COVARIANCE MATRICES UNDER CLUTTER ENVIRONMENT

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Abstract

Clutter has a strong interference on target detection. Different statistical model should be applied to specific situation. In this paper, we propose a function of covariance matrix based algorithm to detect target under clutter environment. The statistical covariance matrices with and without target are usually different, thus we can find the target present or absent. The advantage of proposed algorithm is it works effectively with limited sample data when target signal strength is very weak. The probabilities of detection versus SNR and ROC curves using sinusoidal target on Rayleigh distribution clutter model, log-normal distribution clutter model, and Weibull distribution clutter model show that the algorithm is valid for different clutter models.

1 Introduction

An increasing interest has been directed towards the design of target detection under non-Gaussian, clutter-dominated disturbance. Clutter is unwanted echoes from natural environments, which includes echoes from land, sea, weather, and animals. In order to quantify the effect of clutter on the probability of detection, we must first specify sets of models suitable for representing the clutter [12].

Clutter can be quite varied, so many models and models with numerous parameters will be needed to represent the varieties of clutter. Many distributions provide state and variance estimates, and the correct model is one that captures a majority of the data. For instance, a commonly used density model for clutter is chi-square for power, or equivalently, Rayleigh for amplitude. However, log-normal and Weibull distribution have proved to be better suited for the clutter in some situation [11]. Besides, combining distribution of data as mixture model has been proposed to complement the single parameter model [2].

A number of sample covariance matrix based space-time adaptive processing (STAP) detectors [1, 5] and parameter STAP detectors [10] have been proposed. Various generalized likelihood ratio test (GLRT) based methods [3, 8], and parametric GLRT method [13] is an addition to the STAP family. Given the subspace of the target, the corresponding subspace obtained from the received signal should show large similarity with the given subspace assuming the target is present [4]. The similarity matching is executed in both linear subspace and kernel subspace.

However, it is difficult to solve the detection problem under extremely low signal-to-noise ratio (SNR), like -30 dB. To circumvent this difficulty, function of matrix based detection (FMD) algorithm will be employed for target detection under clutter environment. Based on the matrix inequality, if the target signal are included in the received signal, the trace operation on function of received covariance matrix will always larger than the function of covariance matrix without target signal. Thus we can find a threshold to detect the existence of target signal. The proposed algorithm is blind, which does not require the prior knowledge of structure of either target, noise or clutter. Meanwhile, the algorithm is valid to any kind of clutter model, only some performance differences between different models.

The organization of this paper is as follows. In section 2, system models are described including sample covariance matrix based binary hypothesis test model and statistical clutter model. Function of covariance matrix based detection algorithm in collaborative sensing scenario is proposed in section 3. The experimental results using simulated sinusoidal signal as target signal in Rayleigh distribution, log-normal distribution, and Weibull distribution clutter models are shown in section 4. Finally, the paper is concluded in section 5.

2 System Model

2.1 Binary Hypothesis Test

Let $x[n]$ be the received signal samples after unknown channel. There are two hypotheses to detect target’s existence, $H_0$, target does not exist; and $H_1$, target exists. The received signal samples under the two hypotheses are given respectively as follows:

$$H_0 : x[n] = c[n] + w[n]$$ (1)

$$H_1 : x[n] = s[n] + c[n] + w[n]$$ (2)

...
where $c[n]$ is the received clutter disturbance, which follows different distributions in different environments. $w[n]$ is the received white noise, and each sample of $w[n]$ is assumed to be independent identical distribution (iid), with zero mean and variance $\sigma^2_w$. $s[n]$ is the received target samples after unknown channel with unknown signal distribution. And, it is assumed that clutter and target, noise and target are all uncorrelated. Though in practice, the noise $w[n]$ after analog-to-digital (ADC) is usually non-white, we can use pre-whitening techniques to whiten the noise samples. In the rest of this paper, all noise is considered white.

Two probabilities of interest to evaluate detection performance. One is probability of detection $P_d$, that is, at hypothesis $H_1$, the probability having detected the target. The other is probability of false alarm $P_a$, the probability having detected the target at hypothesis $H_0$.

Assume target signal detection is performed based on the statistics of the $i$th moment sensing segment $\Gamma_{x,i}$, which consists of $N_s$ sensing vectors with $L$ (called "smoothing factor") consecutive output samples in each vector:

$$\Gamma_{x,i} = \{x_{i(1)}N_s+1, x_{i(1)}N_s+2, \cdots, x_{i(1)}N_s+N_s\}$$ (3)

$$x_i = [x[i], x[i+1], \cdots, x[i+L-1]]^T$$ (4)

where $x_i \sim N(\mu_x, R_x)$. If $N_s$ is large enough, $R_x$ can be approximated by sample covariance matrix $\hat{R}_x$:

$$\hat{R}_x = \frac{1}{N_s} \sum_{i=1}^{N_s} (x_i - \mu_x) (x_i - \mu_x)^T$$ (5)

We will use $\hat{R}_x$ instead of $R_x$ for convenience. Accordingly, we have $R_s$ for $s_i$, $R_c$ for $c_i$, and $R_n$ for $w_i$. As we know, $R_s$ is a positive semi-definite matrix with low rank, while $R_c$ and $R_n$ is a positive definite matrix.

Similarly, the cross-correlation between signal and clutter is defined as

$$R_{sc} = \frac{1}{N_s} \sum_{i=1}^{N_s} (s_i - \mu_s) (c_i - \mu_c)^T$$ (6)

$$R_{cs} = \frac{1}{N_s} \sum_{i=1}^{N_s} (c_i - \mu_c) (s_i - \mu_s)^T$$ (7)

Accordingly, $R_{sc}$ and $R_{cs}$ for cross-correlation between signal and noise. $R_{cs}$ and $R_{nc}$ for cross-correlation between clutter and noise.

If target signal is contained in received signal, then we have:

$$R_x = R_s + R_c + R_n + R_{sc} + R_{cs} + R_{sn} + R_{cn} + R_{nc}$$ (8)

Since the number of received data is very huge, we assume the target signal and clutter, target signal and noise to be uncorrelated, which means $R_{sc} = R_{cs} = R_{sn} = R_{cs} = 0$, then the two hypotheses can be recast as

$$H_0 : R_x = R_c + R_n + R_{cn} + R_{nc}$$ (9)

$$H_1 : R_x = R_s + R_c + R_n + R_{sc} + R_{cs} + R_{sn} + R_{cn} + R_{nc}$$ (10)

The detection will be based upon the covariance matrix of received signal $R_x$ in sensing segment $\Gamma_{x,i}$.

However, the assumption about target and noise to be uncorrelated is just for theoretical analysis. In the simulation, all the cross-correlation elements will be contained in the received signal.

2.2 Clutter Statistical Model

Since clutter within a resolution cell or volume is composed of a large number of scatters with random phases and amplitude, it is statistically described by a probability distribution function. The type of distribution depends on the nature of clutter itself (sea, land, volume), and the grazing angle [9].

If sea or land clutter is composed of many small scatterers when the probability of receiving an echo from one scatterer is statistically independent of the echo received from another scatterer, then the clutter maybe modeled using a Rayleigh distribution,

$$f(x) = \frac{2x}{x_0} e^{-\frac{x^2}{2x_0^2}}, x \geq 0$$ (11)

where $x_0$ is the mean-square value of $x$.

The log-normal distribution best describes land clutter at low grazing angles. It also fits sea clutter in the plateau region. It is given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi} x} e^{-\frac{(\ln x - \ln x_m)^2}{2\sigma^2}}, x > 0$$ (12)

where $x_m$ is the median of the random variable $x$, and $\sigma$ is the standard deviation of the random variable $\ln(x)$.

The Weibull distribution is used to model clutter at low grazing angles, usually less than 5 degrees, for frequencies between 1 and 10 GHz. The Weibull probability density function is determined by the Weibull slope parameter $\alpha$ and a median scatter coefficient $\sigma_0$, which is given by

$$f(x) = \frac{b x^{b-1}}{\sigma_0} e^{-\left(\frac{x}{\sigma_0}\right)^b}, x \geq 0$$ (13)

where $b = 1/\alpha$ is known as the shape parameter. Note that when $b = 2$ the Weibull distribution becomes a Rayleigh distribution.

3 Collaborative Detection Algorithm

In this section, we introduce the function of matrix based detection algorithm in collaborative sensing scenario. In this scenario, each sensor node does not need to have powerful computing capability and it only needs several hundreds of data to calculate the covariance matrix simultaneously, meanwhile the best performance could be achieved. Then the mediate results are sent to and combined in fusion center for a final decision, as shown in Fig. 1. It costs less time for calculation and has a faster response compared with single sensing scenario.
Suppose there are $K$ sensors present in the sensor network. The $i^{th}$ moment sensing segment of $k^{th}$ sensor can be represented from Eq. (3), (4)

\[
\mathbf{\Gamma}_{x,k,i} = \{x_{k,(i-1)N_s+1}, x_{k,(i-1)N_s+2} \cdots, x_{k,(i-1)N_s+N_s}\}
\]

\[
x_{k,i} = [x_k[i], x_k[i+1], \cdots, x_k[i+L-1]]^T
\]

where $x_{k,i} \sim \mathcal{N}(\mu_{x,k}, \mathbf{R}_{x,k})$. Similarly, each sensor calculates the covariance matrix $\mathbf{R}_{x,k}$, which can be obtained as,

\[
\mathbf{R}_{x,k} = \frac{1}{N_s} \sum_{i=1}^{N_s} (x_{k,i} - \mu_{x,k})(x_{k,i} - \mu_{x,k})^T
\]

Accordingly, we have target signal covariance matrix $\mathbf{R}_{s,k}$, clutter covariance matrix $\mathbf{R}_{c,k}$ and noise covariance matrix $\mathbf{R}_{n,k}$. $\mathbf{R}_{c,n,k}$ and $\mathbf{R}_{n,c,k}$ are the cross-correlation between clutter and noise. Similar to Eq. (10), if original signal is contained, we have

\[
\mathbf{R}_{x,k} = \mathbf{R}_{s,k} + \mathbf{R}_{c,k} + \mathbf{R}_{n,k} + \mathbf{R}_{c,n,k} + \mathbf{R}_{n,c,k}
\]

Because $\mathbf{R}_{s,k}$ is positive semi-definite matrix, for all $k$

\[
\mathbf{R}_{s,k} + \mathbf{R}_{c,k} + \mathbf{R}_{n,k} + \mathbf{R}_{c,n,k} + \mathbf{R}_{n,c,k} \
\succ \
\mathbf{R}_{c,k} + \mathbf{R}_{n,k} + \mathbf{R}_{c,n,k} + \mathbf{R}_{n,c,k}
\]

We can make $\mathbf{R}_{s,n,k} = \mathbf{R}_{c,k} + \mathbf{R}_{n,k} + \mathbf{R}_{c,n,k} + \mathbf{R}_{n,c,k}$, then the above equation can be simplified as

\[
\mathbf{R}_{s,k} + \mathbf{R}_{c,n,k} \succ \mathbf{R}_{n,c,k}
\]

Then we can use trace and an appropriate monotonically increasing rational function $f$ to both sides of the above inequality to transform the matrix into a number for final comparison.

\[
\text{Tr} (f(\mathbf{R}_{s,k} + \mathbf{R}_{c,n,k})) > \text{Tr} (f(\mathbf{R}_{n,c,k}))
\]

The values after trace calculation from each sensor are sent to the fusion center. The fusion center will compute the average of all $K$ received values, which satisfies

\[
\frac{1}{K} \sum_{k=1}^{K} \text{Tr} (f(\mathbf{R}_{s,k} + \mathbf{R}_{c,n,k})) > \frac{1}{K} \sum_{k=1}^{K} \text{Tr} (f(\mathbf{R}_{n,c,k}))
\]

Hence, we propose detection algorithm 1 in collaborative sensing scenario, namely, FMD in collaborative sensing scenario.

**Algorithm 1 FMD in collaborative sensing scenario**

1: Decide the probability of false alarm $P_{fa}$ and threshold $\gamma$ according to system requirement.
2: Calculate covariance matrix of each sensor.
3: Find an appropriate rational function $f$ and get the function $f$ result of covariance matrix.
4: Obtain the trace of the function of covariance matrix.
5: Collect all the trace results together and compute the average value as a metric $\varphi$

\[
\varphi = \frac{1}{K} \sum_{k=1}^{K} \text{Tr} (f(\mathbf{R}_{x,k}))
\]

6: if $\varphi > \gamma$ then
7: Target exists
8: else
9: Target does not exist
10: end if

4 Simulation Results and Discussion

In the following, we will give some simulation results using random generated sinusoidal signal. For simplicity, we choose monotonically increasing rational function $f(x) = x$, though other monotonically increasing rational functions are also can be applied into this algorithm. For each simulation, zero-mean i.i.d. Gaussian noise and clutter signal are added. 1,000 simulations are performed on each SNR level.

The signal received from target assumes to be the sum of three sinusoidal functions with unit amplitude of each. Assume each sensor needs 600 sample data to calculate covariance matrix, and there are 166 sensors in the network. The number of total sinusoidal samples including all sensors approximately equals to 100,000. The smoothing factor $L$ is chosen to be 8. Probability of false alarm is fixed with $P_{fa} = 10\%$.

We consider three clutter statistical models in simulation as introduced in Subsection 2.2. In Rayleigh distribution, the mean square value $x_0$ has a significant impact on the performance. We change the $x_0$ for each simulation and compare the probability of detection of FMD with different $x_0$ in Fig. 2. When $x_0 = 1$, FMD can achieve 0.5 probability of detection at SNR -34 dB. We can see this curve almost overlaps with the curve without clutter which achieves 0.5 probability of detection at SNR -34 dB. The analysis of performance without clutter but noise can be found in [6,7]. As $x_0$ increases, the clutter became more and more dominant in the received signal, which results in the decline of the probability of target signal detection. However, even when $x_0 = 64$, FMD still can get 0.5 probability of detection at SNR -27.5 dB.

In log-normal distribution, two parameters can completely describe the distribution. One is $\mu$, mean of $\ln(x)$. The other is $\sigma$, standard deviation of $\ln(x)$. The probability of detection with different parameters are compared in follows. When $\sigma$ is fixed to be 1, from Fig. 3, tiny change of mean value $\mu$ has a significant impact on the performance. When $\mu = 0$, 0.5 probability of detection is reached at -31 dB. Similarly, we can get -27.5
Probability of Detection with Different Mean Square Value

Figure 2: Probabilities of detection among different mean square values in Rayleigh distribution clutter model

Probability of Detection with Different Standard Derivation Value

Figure 4: Probability of detection comparison among different standard deviations in log-normal distribution clutter model

Probability of Detection with Different Mean Value

Figure 3: Probability of detection comparison among different mean values in log-normal distribution clutter model

Probability of Detection with Different Parameters Combination

Figure 5: Probabilities of detection comparison among different parameters combination in Weibull distribution clutter model

dB when \( \mu = 0.5 \), and -22.5 dB when \( \mu = 1 \). When \( \mu \) changes from 0 to 1, the gain lost is 8.5 dB. Here we see the influence of \( \sigma \) in Fig. 4, with \( \mu \) set to be 0. The performance with \( \sigma = 0.5 \) only declines a little compared with which \( \sigma = 0 \). When \( \sigma \) increases up to 1.5, the SNR for FMD to reach 0.5 probability of detection is only -21 dB. Though \( \sigma \) varies linearly, the gain lost is nonlinear because of the natural logarithm \( \ln \) operation.

The detection performance with Weibull distribution clutter model is provided in Fig. 5. Because the Weibull distribution becomes Rayleigh distribution if the shape parameter \( b = 2 \), we set \( b \) to be 1 and 3. When \( b = 3 \) and \( x_0 = 8 \), we can reach the best performance among other parameters combinations, which is close the performance without clutter shown in Fig. 2.

Receiver operating characteristic (ROC) curves are provided for the three clutter models using some typical parameter settings. The initial probability of false alarm is 0.001, and increases with step 0.05. The ROC curves obtained at SNR -30 dB and -25 dB are shown in Fig. 6 and Fig. 7. When SNR is at -20 dB, the proposed algorithm can achieve a high probability of detection at a low probability of false alarm. With all the clutter models, the algorithm can reach more than 50% probability of detection at 0.001 probability of false alarm, and 100% probability of detection less than 0.5 probability of false alarm. The scale is shrunk in Fig. 8 to make the curves easier to see, but three curves are still overlapped at 100% probability of detection.

In addition, the performance can be improved further if we use more sensors. This is because more samples can make the numerical difference between two hypotheses more stable, hence the threshold can totally separate signals from two hypotheses. Although we assume signal and clutter, signal and noise are
In some specific situations, there is not so many sensors can be deployed. Can this algorithm still perform well with less sensors? The answer is positive. For example, we only have 10 sensors. For fair comparison, the number of total samples including all sensors is also set 100,000, then each sensor needs to process 10,000 sample data. Inside of each sensor, we divide the 10,000 data into 16 subsegment, with each segment contains 600 samples. After obtaining the function of matrices results of each subsegment, we add them together and average, then apply Trace operator to get the metric for one single sensor. As we known Trace operator has the property that

$$\text{Tr} (B + C) = \text{Tr} (B) + \text{Tr} (C)$$

We can get

$$\frac{1}{K} \sum_{k=1}^{K} \text{Tr} (f (R_{x,k})) = \frac{1}{K} \sum_{k=1}^{K} \left( \text{Tr} \left( \frac{1}{J} \sum_{j=1}^{J} f (R_{x,j}) \right) \right)$$

if $K = J \times \tilde{K}$. Where $J$ is the number of the subsegments in each sensor. The right side of Eq. (24) describes how the detection metric can be obtained with multiple sensors, denoted as $\tilde{K}$ sensors, and multiple subsegments, denoted as $J$ subsegments, to be processed inside one sensor.

We conclude that the detection performance involving with different numbers of sensors are similar in some degree. The only difference is that if more data need to be processed by one sensor, the sensors require strong calculation capacity to provide result immediately, because several function of matrices calculations and matrices additions need to be performed. The computation complexity depends on the matrix function we choose. While if sensors are enough, each sensor only needs to calculate function of matrix once without involving any matrices addition.

5 Conclusion

In this paper we considered the weak target signal detection under clutter environment. Sample covariance matrices were calculated based on received signals. Trace operation on sample covariance matrices are different between the situations that target signal present and absent. One function of matrix based detection algorithm was proposed in collaborative sensing scenario. Simulated sinusoidal signal was employed as target signal in simulation. The simulation results shown that the algorithm work well on all clutter distribution models. When the
clutter strength is small, the performance is almost the performance of detection without clutter. Our future research direction includes achieving similar performance with less data.

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