UWB Pulse Propagation and Detection

0.1 INTRODUCTION

Radio propagation is essential to wireless communications [16]. When a short pulse is used for a UWB system [1], some unique features occur [2]. When a short UWB pulse propagates through a channel, multiple pulses are received via multipath. This is true for both narrowband and UWB. However, unlike narrowband systems, for UWB case these pulses in general have pulse shapes different from the incident short pulse. This phenomenon is called pulse distortion. Pulse distortion can be caused by frequency dependency of the propagation channel and antennas. The per-path impulse response is introduced to describe pulse distortion for each individual path. The impact of pulse distortion on the baseband transmission has been investigated in the past [2-10]. This chapter is intended to address these issues related to pulse signal detection of distorted pulses.

0.2 UWB PULSE PROPAGATION

0.2.1 GENERALIZED MULTIPATH MODEL

Based on the Geometric Theory of Diffraction (GTD) framework and principle of locality, we establish the fact that the total response from a complex multipath channel is modeled by the sum of the impulse responses of local scattering centers [6,8,9]:

\[ h(\tau, \theta, \varphi) = \sum_{n=1}^{N_{GO}} a_n(\theta, \varphi) \delta(\tau - \tau_n) + \sum_{n=1}^{N_d} b_n(\tau, \theta, \varphi) * \delta(\tau - \tau_n) \]  

(0.1)

where \( N_{GO} \) and \( N_d \) represent the number of geometric optics rays and diffracted rays, respectively. The operation \(*\) denotes convolution in (0.1). The transient responses of the diffracted rays (the 2\(^{nd}\) term) can be obtained through exact, experimental, numerical and asymptotic methods. A geometric optics ray can be treated as a generalized diffracted ray with \( h_n(\tau, \theta, \varphi) = \delta(\tau, \theta, \varphi) \). In general the per path impulse response \( h_n(\tau, \theta, \varphi) \) are obtained from the solutions of the time-domain Maxwell equations. It is analytically impossible to give the solution for an arbitrary generalized diffracted ray that is valid for arbitrary configurations. Rather we are interested the expressions valid for in a large class of such rays. These expressions are sufficient for most of configurations in our engineering applications. Fortunately we can define the field in the neighborhood of the singularity \( \tau = \tau_n \) only. The field
component of a generalized ray has the behavior

\[ h_n(\tau) = \begin{cases} 
\xi(\tau - \tau_0) \sum_{n=0}^{\infty} \frac{C_n}{n!} (\tau - \tau_0)^n, & \tau < \tau_0 \\
\eta(\tau - \tau_0) \sum_{n=0}^{\infty} \frac{D_n}{n!} (\tau - \tau_0)^n, & \tau > \tau_0 
\end{cases} \]

then the corresponding transfer function has the form \[6,8,9\]

\[ H_n(\omega) = \sum_{n=0}^{\infty} \left\{ \frac{D_n}{n!} \frac{1}{(j\omega)^n} \int_0^\infty \eta(t) \frac{t}{j\omega} t^n e^{-t} dt - \frac{C_n}{n!} \frac{1}{(-j\omega)^n} \int_0^\infty \xi(-t/j\omega) t^n e^{-t} dt \right\} \]

(0.2)

where \( \xi(\tau) \) and \( \eta(\tau) \) are rather general and need definition for a specific configuration. Note that these two functions are only dependent on the local properties of a ray such as the edge of a building. For the important special case in which both \( \xi(\tau) \) and \( \eta(\tau) \) are \( \tau^{1/2} \), the asymptotic series contains fractional powers of \( 1/\omega \), e.g., \( 1/\omega^{n+1/2} \) where \( n \) is an integer. For \( \omega \to \infty \) each term of the series vanishes. This is as expected, for there is no classical diffracted geometric optics field. The leading term for large \( \omega \) may well serve as the geometric optics diffracted field. For a special case with an integer \( n \), via inverse Laplace transform we obtain

\[ h_n(\tau) = \begin{cases} 
\sum_{n=0}^{\infty} \frac{D_n}{n!} (\tau - \tau_0)^n, & \tau > \tau_0 \\
0, & \tau < \tau_0 
\end{cases} \]

\[ H_n(\omega) = \sum_{n=0}^{\infty} \frac{D_n}{(j\omega)^n} \Gamma(n + \frac{3}{2}) \]

(0.3)

Eq. (0.3) is valid a special class of rays. All the rays diffracted by a single wedge and multiply diffracted by several wedges can be modeled by (0.3). This model is sufficiently general for commonly encountered shapes predicted by GTD or UTD. Heuristically we can postulate that a more generalized class of rays has the per path frequency response of \( (j\omega)^{\alpha_n} \), where \( \alpha_n \) is a real number. This generalization was first postulated one decade ago \[11,12\]. If a slow-varying frequency response \( H_n(\omega) \) can be fitted approximately by the curve of \( (j\omega)^{\alpha_n} \) for the \( n \)-th path, this postulation can be assumed valid. The justification of this postulation is its mathematical simplicity.

Based on the above postulation the generalized model of a form can be expressed as

\[ H(\omega) = \sum_{n=1}^{N} A_n (j\omega)^{\alpha_n} e^{j\omega n} \]

\[ h(\tau) = \sum_{n=1}^{N} \frac{A_n}{\Gamma(-\alpha_n)} \tau^{-(1+\alpha_n)} U(\tau) * \delta(\tau - \tau_n) \]

(0.4)

where \( \alpha \) is a real value. This mode is asymptotically valid for incident, reflected, singly diffracted and multiple reflected/diffracted ray path field. Our commonly used geometric shapes are given in Table 1. For practical problems of pulses propagating through walls and buildings a more generic function the one given in Eq. (0.4) is needed. Note the parameter \( \alpha_n \) can be a random variable, as first suggested in [12]. We are measuring this function in the lab. The results will be reported elsewhere.
The motivation of using the function in Eq. (0.4) is its simplicity and feasibility for a large category of problems. This form can allow us to use the powerful mathematical tool of fractional calculus [14] to conveniently describe our problems.

### 0.2.2 IEEE 802.15.4a Channel Model

A special form of (0.4) is adopted in the IEEE 802.15.4a channel model [13, 29]. The frequency dependency of each path is assumed to be

\[ H_n(j\omega) = (j\omega)^\alpha \]  

where \( \alpha \) lies between 0.8 and 1.4 (including antennas effects), while excluding antennas effects, \( \alpha \) is found to be between -1.4 (in industrial environments) and +1.5 (in residential environments). The first suggestion of using (0.4) was made in [12]. Alternative modeling of the frequency dependency of the transfer function for each path includes a frequency-dependent pathloss exponent \( n(f) \) and an exponential dependency \( \exp(-\chi f) \), with \( \chi \) between 1.0 (LOS) and 1.4 (NLOS) [29].

### 0.3 UWB Pulse Signal Detection

#### 0.3.1 Optimum Receiver

This section investigates the performance of the mismatched filter in the receiver [3]. For convenience the generalized channel model (0.1) is postulated as

\[ h(\tau) = \sum_{n=1}^{L} A_n h_n(\tau) * \delta(\tau - \tau_n) \]  

where \( L \) generalized paths are associated with amplitude \( A_n \), delay \( \tau_n \), and per-path impulse response \( h_n(\tau) \). The \( h_n(\tau) \) represents an arbitrary function that has finite energy. Eq. (0.6) together with (0.4) is sufficient for most practice use. When a pulse \( p(t) \) passes through the path given by Eq. (0.4), the distorted pulse is given by

\[ y_n(t) = p(t) * h_n(t) = \left( \frac{d}{dt} \right)^{\alpha_n} p(t) \]  

where \( \left( \frac{d}{dt} \right)^{\alpha_n} \) is the fractional differential of the \( p(t) \). Fractional calculus is a powerful tool in calculation and manipulation [14].

One goal of this chapter is to investigate how pulse distortion would affect the system performance if no compensation for this kind of distortion is included. To achieve this, we first present a general system model and give its performance expression. In Fig.0.3.1 we present a general binary baseband system model for the physics-based signals. The signals considered are given by

\[ s_0(t) = A\psi_0(t), \quad s_1(t) = A\psi_1(t) \]
where \( \{ \psi_0, \psi_1 \} \) is a binary signal set. Here \( \psi_0 \) and \( \psi_1 \) are finite-energy, time-limited signals of duration \( T \). (When a pulse \( p_i(t), i = 0, 1 \) is transmitted at the transmitter, the received signal \( s(t) = p(t) * h(t), i = 0, 1 \), where \( h(t) \) is the impulse response of the channel, as given in the next section.)

The received signal \( Y(t) \) is the sum of the noise \( X(t) \) and the signal \( s_i(t) \) where \( i = 0 \) if the binary digit “0” is sent and \( i = 1 \) if the binary digit “1” is sent. The filter shown in Fig. 0.3.1 is a time-invariant linear filter with impulse response \( q(t) \). The output of this filter, which is denoted as \( Z(t) \), is sampled as time \( T_0 \). The output \( Z(T_0) \) of the sampler is then compared with an arbitrary threshold \( \gamma \) in order to make a decision between two alternatives 0 and 1. In Fig.0.3.1, the noise \( X(t) \) is additive Gaussian noise (AGN) channel, not necessarily white. The channel noise is stationary with zero mean and is independent of the input to the receiver. The filter \( q(t) \) is not necessarily matched to the signal \( s_i(t) \). It is the optimization of the \( q(t) \) that motivates this paper.

\[
\begin{align*}
S_0(t) & \xrightarrow{+} Y(t) & Z(t) \\
S_1(t) & \xrightarrow{?} k \\
X(t) & \xrightarrow{q(t)} \text{filter} & i = T_0 \\
\end{align*}
\]

Fig. 0.1 General model for binary baseband data transmission.

Following the steps of [15,3], we can obtain the probability of error. The probability of error when 0 is sent and when 1 is sent is given by, respectively,

\[
P_{e,0} = Q \left( \frac{\mu_0(T_0) - \gamma}{\sigma} \right) \tag{0.9}
\]

\[
P_{e,1} = Q \left( \frac{\gamma - \mu_1(T_0)}{\sigma} \right) \tag{0.10}
\]

where, \( \mu_i(t) = s_i(t) * q(t), i = 0, 1 \), and \( \sigma^2 = (q(t) * q(-t) * R_X(t)) \big|_{t=0} \). \( Q(x) \) is defined as \( Q(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-y^2/2)dy \). \( R_X(t) \) is the auto-correlation of the AGN \( X(t) \). For the AWGN noise, we have \( \sigma^2 = N_0 \int_{-\infty}^{\infty} q^2(t)dt \) where \( N_0 \) is the spectral density for the white noise process \( X(t) \).

\[1\] The finite-energy condition is important. We will find that the received physics-based signals satisfy this condition. Although the signals are singular at some points, their energy is finite.
In Eq. (0.9 0.10), the $\gamma$ is arbitrary. When the Bayes decision criterion is used [15], the threshold is given by

$$\tilde{\gamma} = \frac{\mu_0(T_0) + \mu_1(T_0)}{2} + \frac{\sigma^2 \ln(\pi_1/\pi_0)}{\mu_0(T_0) - \mu_1(T_0)}$$  \hfill (0.11)$$
where $\pi_0$ and $\pi_1$ are probabilities of that 0 and 1 are sent, respectively.

The average probability is thus given by

$$\bar{P}_e = \pi_0 P_e,0(\tilde{\gamma}) + \pi_1 P_e,1(\tilde{\gamma}) = \pi_0 Q[SNR - (2SNR)^{-1} \ln(\pi_1/\pi_0)] + \pi_1 Q[SNR + (2SNR)^{-1} \ln(\pi_1/\pi_0)]$$  \hfill (0.12)

where the SNR is the signal-to-noise ratio at the input to the threshold device, given by

$$SNR = \frac{\mu_0(T_0) - \mu_1(T_0)}{2}$$  \hfill (0.13)$$
When 0 and 1 are sent with equal probability, then $\pi_0 = \pi_1 = 1/2$. As a result, the average probability is reduced to

$$\bar{P}_e = Q(SNR)$$  \hfill (0.14)$$

Now we can optimize $q(t)$ in terms of SNR defined by

$$SNR = \frac{(s_0(t) * q(t))|_{t=T_0} - (s_1(t) * q(t))|_{t=T_0}}{\sqrt{2N_0||q||}}$$  \hfill (0.15)$$
where $||q|| = \left(\int_{-\infty}^{\infty} q^2(t)dt\right)^{1/2}$ is the norm of $q(t)$. $s_0(t) = p_0(t) * h(t)$ and $s_1(t) = p_1(t) * h(t)$ can be singular but their energies are limited, as will be observed in Eq. (0.35).

Eq. (0.12) and Eq. (0.14) are valid when the filter impulse response, $q(t)$, is arbitrary. The receiver optimum filter is known to be the matched filter that is matched to the signal $s_0(t)$ and $s_1(t)$, the pulse waveforms received at the receiver antennas. However, sometimes our receivers are designed to be matched to $p_0(t)$ and $p_1(t)$, the pulse waveforms at the transmitter. This mismatch practice will result in performance degradation in terms of SNR, which will be illustrated by numerical results.

UWB antennas usually distort the transmitted pulse [2]. The transmitter and receiver antennas can be as a whole modeled as a linear filter with impulse response, $h_a(t)$. Their impact can be absorbed in the new transmitted pulse waveforms, $\tilde{p}_0(t) = p_0(t) * h_a(t)$ and $\tilde{p}_1(t) = p_0(t) * h_a(t)$. So the received signals are $\tilde{s}_0(t) = \tilde{p}_0(t) * h(t)$ and $\tilde{s}_1(t) = \tilde{p}_1(t) * h(t)$. To simplify the analysis, we assume $h_a(t) = \delta(t)$. In other words, we ignore the antennas impact at this point and will report elsewhere. In the next section we will derive $h(t)$ in a closed form for a typical configuration. As a
result, the expressions Eqs. (0.14) and (0.15) combined with the closed-form $h(t)$ will be very useful.

### 0.3.2 GENERALIZED RAKE RECEIVER

All the signal processing algorithms require knowledge of the channel parameters in order to detect the signal. The channel must be estimated prior to the actual detection. We use a data-aided (DA) approach [22,23] where the data frame begins with a sequence of known data, so called pilot signal. The generalized RAKE receiver can be used to replace the matched filter in the presence of inter-symbol-interference [7] and multiuser detection [4]. To avoid the channel estimation of the generalized RAKE, we can use the auto-correlation receiver based on Transmitted Reference [31].

#### A. Two-dimensional Tap-delayed Line Model

The key of our generalized RAKE is to use an FIR filter to represent the per-path impulse response $h_n(\tau)$ in Eq. (0.6):

$$h_n(\tau) = \sum_{m=1}^{M} \beta_{mn} \delta(\tau - \tau_{mn}) \quad (0.16)$$

This FIR filter has M taps with tap spacing $T_s$. The received signal is sampled every $T_s$ seconds. Note a FIR representation of pulse distortion was used in channel modeling [24]. As a result we obtain many (say M) discrete taps for each generalized path. Eq. (0.6) is rewritten as

$$h(\tau) = \sum_{n=1}^{P} \sum_{m=1}^{M} \tilde{a}_{mn} \delta(\tau - \tilde{\tau}_{mn}) \quad (0.17)$$

where $\tilde{a}_{mn} = A_n \beta_{mn}$ is the real amplitude of each tap corresponding to $\tau_{mn}$. With a mapping, we can reduce the two-dimensional model to a one-dimensional discrete model

$$h(\tau) = \sum_{l=1}^{L} a_l \delta(\tau - \tilde{\tau}) \quad (0.18)$$

where

$$L = MP$$

$$\tilde{\tau} = \tilde{\tau}_{[m+(n-1)M]} = \tilde{\tau}_{mn},$$

$$a_l = \tilde{a}_{mn} = A_n \beta_{mn},$$

$$l = m + (n-1)M, m = 0, 1, \ldots, M, n = 0, 1, \ldots, P.$$  

The one-dimensional discrete model can be handled using conventional channel estimation algorithm that is used for a narrowband system. Thus the FIR representation reduces our channel estimation for the generalized RAKE to that of the conventional RAKE.

#### B. Optimal ML Channel Estimation
Following steps of [4,22,23], the received signal can be expressed as

$$r = Da + \eta \quad (0.19)$$

where

$$[D]_{jl} = \sum_{i=0}^{N_P-1} b_P(i)g_0^l(jT_s - iT - \tau_l)$$

$\eta$ is AWGN with two-sided spectral density $N_0/2$ and $a$ is the vector of the channel amplitude $a_l$ defined in Eq. (0.18). $g_0^l$ is the spreading waveform with duration $T$. For multiuser detection, $g_0^l$ is given by (0.38). We assume that a frame consists of $N_p$ known pilot symbols $b_P$. The received channel signal $r$ is Gaussian with mean $Da$ and covariance matrix $C$ that has terms of noise variance on its diagonal and zeros elsewhere. The optimal channel estimation is to maximize a function $\Lambda(a, \tau)$ where

$$\Lambda(a, \tau) = 2r'C^{-1}Da - a'D'C^{-1}Da \quad (0.20)$$

where $\tau$ is a vector of channel path delays $\tau_l$ corresponding to amplitudes $a_l$. The search for the optimum is complex and we will use a sub-optimum algorithm in the following.

**C. Successive Channel (SC) Estimation**

We can use the above optimal channel estimation for a one-tap channel. The estimated delay and amplitude are

$$\hat{\tau} = \arg \max \left\{ \left| \frac{\zeta(\tau)C^{-1}r}{\zeta(\tau)C^{-1}\zeta(\tau)} \right|^2 \right\} \quad (0.21)$$

$$\hat{a} = \frac{\zeta(\hat{\tau})C^{-1}r}{\zeta(\hat{\tau})C^{-1}\zeta(\hat{\tau})} \quad (0.22)$$

where

$$[\zeta(\tau)]_m = \sum_{i=0}^{N_P-1} b_P(i)g_0^l(mT_s - iT - \tau), \quad 1 \leq m \leq M$$

where $g_0^l(t)$ has a duration $T$. The above scheme can be performed iteratively for the multipath channel defined in (0.18). The algorithm is summarized by the following four steps in [4,22,23].

1. Initialization:
   set threshold and $c(\tau) = 0$ for $\tau_{\min} \leq \tau \leq \tau_{\max}$;

2. Perform the search for the strongest tap $\hat{\tau}$ and calculate $\hat{a}$ by using the above equations,

   $$c(\hat{\tau}) \leftarrow c(\hat{\tau}) + \hat{a} \hat{\xi}(\hat{\tau}),$$

   $$r \leftarrow r - \hat{a} \hat{\xi}(\hat{\tau});$$
3. If $\hat{\alpha} \geq \text{threshold}$, go to step 2; otherwise set $\hat{h}(\tau) = c(\tau)$ and stop.

Using the above successive channel estimation algorithm the channel impulse response $\hat{h}(\tau)$ is obtained.

With Eqs. (0.16) and (0.17), the FIR representation of the per-path impulse response is estimated as $\hat{h}_n(\tau)$. For the $n$-th path, the pulse waveform is $q_n(\tau) = p(\tau) \ast \hat{h}_n(\tau)$. Let us consider two cases:

1. If one tap is used in Eq. (0.16), $\hat{h}_n(\tau) = \beta_1 n \delta(\tau - \hat{\tau}_1)$, and thus $q_n(\tau) = \beta_1 n p(\tau - \hat{\tau}_1)$. So the matched filter can be implemented with an impulse response of $p(\tau)$, the transmitted pulse waveform. This special case is just the conventional RAKE receiver used in narrowband and UWB scenario.

2. If several taps (say three) are used in Eq. (0.16), then $\hat{h}_n(\tau) = M \sum_{m=1}^{M} \hat{\beta}_{mn} \delta(\tau - \hat{\tau}_{mn})$ and thus the received pulse waveform for the $n$-th path is estimated as

$$\hat{q}_n(\tau) = M \sum_{m=1}^{M} \hat{\beta}_{mn} p(\tau - \hat{\tau}_{mn})$$

(0.23)

The matched filter for each user should be designed to match $\hat{q}_n(\tau)$, instead of $p(\tau)$. In other words, the front-end filter impulse response is equal to $\hat{q}_n(\tau)$, not $p(\tau)$. The generalized RAKE structure is obtained here.

0.3.3 OPTIMUM RECEIVER WITH INTER-SYMBOL INTERFERENCE

We will incorporate the per-path impulse response into the optimal receiver when ISI is present [2,7]. In absence of pulse distortion, the work in presence of ISI is done in [26,27]. In the following framework, we pay special attention to the $h_n(\tau)$.

For Pulse Amplitude Modulation (PAM) signals in a single-user system, the transmitted signal is expressed as

$$s(t) = \sum_{n=-\infty}^{\infty} b_n x(t - nT_s)$$

(0.24)

where $b_n$ represents the $n$-th discrete information symbol with duration of $T_s$. The received signals are represented as

$$r(t) = \sum_{n=0}^{\infty} b_n y(t - nT_s) + n(t)$$

(0.25)

where $n(t)$ is AWGN. It has been shown for a narrowband system [18] that the optimum receiver structure is illustrated in Fig. 0.3.3.

The received signal $r(t)$ first passes the matched filter followed by a sampler of sampling rate of $1/T_s$. The sampled sequence is further processed by a maximum likelihood sequential estimator (MLSE) detector that was first studied by Forney (1972). This receiver structure is optimal in terms of minimizing the probability of transmission error in detecting the information sequence. If in the UWB receiver
design we assume that $y(t)$ is of finite energy, the structure of Fig.0.3.3 can immediately be applied to our UWB problem [2,7]. The output of the matched filter is expressed as

$$q(t) = \sum_{n=0}^{\infty} b_n R_{yy}(t - nT_s) + v(t)$$  \hspace{1cm} (0.26)

where

$$R_{yy}(t) = y(t) \otimes y^*(t) = R_{xx}(t) \otimes h(t) \otimes h^*(-t)$$  \hspace{1cm} (0.27)

is the autocorrelation of $y(t)$ and $v(t)$ is the response of the matched filter to AWGN noise $n(t)$. $R_{xx}(t)$ is the autocorrelation of $x(t)$. Denote

$$R_{k-n} = R_{yy}(t - nT_s)|_{t=kT_s}$$  \hspace{1cm} (0.28)

When the matched filter $y^*(-t)$ is replaced by a generalized RAKE receiver described above, the performance will be degraded. When this is done, the rest of detection remains the same.

After the sampler in Fig.0.3.3, the discrete signals at times $t = kT_s$ of (0.26) are given by

$$q_k = \sum_{n=0}^{\infty} b_n R_{k-n} + v_k, k = 0, 1, 2, \ldots$$  \hspace{1cm} (0.29)

$R_0$ (the energy of the n-th symbol) can be regarded as an arbitrary scale factor and conveniently set to 1, from (0.29) we obtain

$$q_k = b_k + \sum_{n=0, n\neq k}^{\infty} b_n R_{k-n} + v_k$$  \hspace{1cm} (0.30)

where $b_k$ term represents the expected information symbol of the $k$-th sampling period, the $2^{nd}$ term is the ISI, and $v_k$ is the additive Gaussian variable at the $k$-th sampling point. The sequence of $q_k$ will be further processed by MLSE (Viterbi algorithm) before being sent to decision circuits.

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Fig. 0.2  Optimal receiver structure in presence of ISI in an AWGN channel.
If the duration of each symbol is smaller than the multipath spread $T_D$, i.e., $T_s > T_D$, there is no ISI, thus $R_{k-n} = 0\text{ for } n \neq k$. Then (0.30) reduces to

$$q_k = b_k + v_k$$

(0.31)

It is, therefore, sufficient to detect the symbols independently one-by-one for each given instant $t = kT_s$. This is the famous matched filter for isolated symbol detection. As expected, once matched, the detection depends on the energy $R_0 = R_{yy}(0)$ of the symbol, not the received composite pulse waveform $y(t)$. Thus, per-path pulse distortion has no impact on the single symbol matched filter detector. However, according to (0.30), we will find that this is not the case when ISI is present. We will evaluate this effect using some closed-form expressions for system performance.

If the number of the overlapping symbols is less than ten, the MLSE (Fig.0.3.3) may be even feasible for the state-of-the-art signal processing capability.

**Sub-Optimum Detection of Physics-based Signals**

Let us simplify optimum receiver structures in two directions: simplifying matched filter and sub-optimum decisions. First the matched filter can be simplified in a sub-optimum manner. Since the per-path pulse waveform has been included in the channel impulse, the resultant matched filter is sometimes called “Generalized RAKE” receiver structure [4,5]. When per-path pulse distortion is included, the suboptimum implementation of the matched filter is the RAKE receiver structure. In absence of per-path pulse distortion, the matched filter is identical to the RAKE receiver of Price & Green (1958). Different sub-optimum structures can be used to approximate the front end or the mathematical operation of $R_{yy}(t)$ in Eq. (0.29) [26,27,31].

Let us follow Kailith and Poor [25] and restrict ourselves to a linear detector with a matched filter front end. The detected bits are

$$\hat{b}_n = \text{sign}(z_n)$$

(0.32)

where $z = My$, matrix $M$ is arbitrary, and vector $y$ has elements of $y(t)|_{t=nT_s}$. Further we assume $b_n$ to take on the values of $\pm 1$ with equal probabilities. For zero-forcing, the probability that $b_n$ is in error is simply

$$P_e(n) = Q\left(\frac{\sigma_n}{\sigma\sqrt{(R^{-1})_{n,n}}}\right)$$

(0.33)

where $\sigma$ is the noise density, matrix $R = [R_{k-n}]$ defined in (0.29), and $\sigma_n^2$ is the energy of $y(t - nT_s)$.

Under some general conditions, the error probability of the linear MMSE detector

$$P_c(n) \approx Q\left(\frac{\sigma_n M_{n,n}}{\sigma \sqrt{(M RM)_{n,n}}}\right)$$

(0.34)

where $M = (R + \sigma^2 D_{a}^{-2})^{-1}$ and $D_a = \text{diag}(a_n)$. 
It is observed from (0.33) and (0.34) that per-path pulse distortion affects the system performance through the output of the matched filter. The time-domain overlapping of distorted pulses make the matrix \( R \) non-diagonal. When \( R \) is diagonal, the per-path pulse distortion affects each simple energy separately. Inter-pulse overlapping combined with per-path pulse distortion makes receiver design very challenging.

To gain some insights, let us consider a two-symbol case. Denote

\[
R^{-1} = \begin{bmatrix} 1 & \rho \\ 1 & 1 - \rho^2 \end{bmatrix}^{-1} = \frac{1}{1 - \rho^2} \begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}
\]

where \( \rho = R_1 = R_{-1} = R_{yy}(\tau)|_{\tau = \pm T_s} \leq 1. \)

In (0.33) the performance only depends on the diagonal elements of \( R^{-1} \), which are inversely proportional to \( 1 - \rho^2 \). Compared with the isolated single symbol detector, the performance loss in SNR for zero-forcing detector can be defined as

\[
\beta_n = \frac{1}{\langle R^{-1} \rangle_{n,n}}
\]

and \( \beta_n = 1 - \rho^2 \) for two symbol case. Thus \( \beta_n \) depends on per-path pulse distortion \( h_n(\tau) \) through \( R_{yy}(\tau) \) in (0.35) and (0.29). The \( \beta_n \) depends on the whole physics-based channel model and symbol rate.

### 0.3.4 RECEIVER WITH TIME-REVERSAL CHANNEL IMPULSE RESPONSE

The optimum receiver structure of Fig.0.3.3 is used as a heuristic approach to derive some new structures. In Fig.0.3.4, we show the auto-correlation of the channel impulse response, \( R_{hh}(t) = h(t) \otimes h(-t) \), for an typical IEEE 802.15.3a model. The \( R_{hh}(t) \) is normalized by \( R_{hh}(0) \). The plot shows \( R_{hh}(t)/R_{hh}(0) \). It is interesting to notice that the first peak has an energy that is about 50%. We can look at each pulse as a chip for a spread-spectrum system, and the channel impulse response can be regarded as a pseudo-noise code where the width of multipath delay spread corresponds to the length of the PN code. This observation first made in [30] intuitively suggests that it is more simple to base the detection on \( R_{hh}(t) \), rather than the noisy \( h(t) \).

As given in (0.29), the output of the matched filter is

\[
R_{yy}(t) = R_{xx}(t) \otimes R_{hh}(t)
\]

\( R_{xx}(t) \) is typically of a smooth shape. Finally, a time-reversal scheme is used to shift the design complexity from the receiver to the transmitter [30]. In time-reversal a signal is precoded such that it focuses both in time and in space at a particular receiver. Due to temporal focusing, the received power is concentrated within a few taps and the task of equalizer design becomes much simpler than without focusing.

If we use the pre-filter (at the transmitter) which has an impulse response of \( h(t) \), the output of the matched filter (at the receiver) is \( R_{yy}(t) = R_{xx}(t) \otimes R_{hh}(t) \). If the first tap is used (shown in Fig.0.3.4), the collected energy is about 50% of the total
energy. Implementation of the pre-filter can be simplified using the mono-bit A/D technology [27].

0.3.5 OPTIMUM RECEIVER WITH MULTI-USER DETECTION

Let us consider a DS-CDMA UWB channel that is shared by K simultaneous users. Each user is assigned a signature waveform $g_k^0(t)$ of duration $T$, where $T$ is the symbol interval. A transmitted signature waveform for the $k$-th user may be expressed as

$$g_k^0(t) = p(t) \ast \sum_{n=0}^{N_c-1} c_k(n) \delta(t - nT_c), \quad 0 \leq t \leq T$$  \hspace{1cm} (0.38)

where $\{c_k(n), 0 \leq n \leq N_c - 1\}$ is a pseudonoise (PN) code sequence consisting of $N_c$ chips that take values $\{\pm 1\}$, $p(t)$ is a pulse of duration $T_c$, the chip interval.

Without loss of generality, we assume that K signature waveforms have unit energy. For simplicity we assume that binary antipodal signals are used to transmit the information from each user. Consider a block of N consecutive bits for each user in an observation window. Let the information sequence of the $k$-th user be denoted by $b_k(m)$, where the value of each information bit may be $\pm 1$. It is convenient to consider the transmission of a block of some arbitrary length, say N. The data block from the $k$-th user is

$$b_k = \sqrt{E_k} [b_k(1) \cdots b_k(N)]^T$$  \hspace{1cm} (0.39)

where $E_k$ is the transmitted energy of the $k$-th user for each bit. The transmitted waveform is

$$x_k(t) = \sqrt{E_k} \sum_{i=1}^{N} b_k(i) g_k^0(t - iT)$$  \hspace{1cm} (0.40)
The composite transmitted signal for the $K$ users is

$$x(t) = \sum_{k=1}^{K} x_k(t - T_k) = \sum_{k=1}^{K} \sqrt{E_k} \sum_{i=1}^{N} b_k(i) g_0^k(t - iT - T_k)$$  \hspace{1cm} (0.41)$$

where $T_k$ are the transmission delays, which satisfy the condition $0 \leq T_k \leq T$ for $1 \leq k \leq K$. Without loss of generality, we assume that $0 \leq T_1 \leq T_2 \leq \cdots \leq T_K < T$. This is the model in an asynchronous mode. For synchronous mode, $T_k = 0$ for $1 \leq k \leq K$. We assume that the receiver knows $T_k$.

**Fig. 0.4** Generalized RAKE for multiuser detection. Estimated channel impulse response is used in forming the signature waveform $g_k(t)$ for the $k$-th user.

At the receiver end, the corresponding equivalent low-pass, received waveform may be expressed as

$$r(t) = y(t) + n(t)$$  \hspace{1cm} (0.42)$$

where $n(t)$ is AWGN, with power spectral density of $\frac{1}{2}N_0$. The received signal is

$$y(t) = \sqrt{E_k} \sum_{k=1}^{K} \sum_{i=1}^{N} b_k(i) g_k(t - iT - T_k)$$  \hspace{1cm} (0.43)$$

where $g_k(t)$ is the received signature waveform given by

$$g_k(t) = g_0^k(t) * h^{(k)}(t) = \left[p(t) * h^{(k)}(t)\right] * \sum_{n=0}^{N_c-1} c_k(n) \delta(t - nT_c)$$  \hspace{1cm} (0.44)$$

where $h^{(k)}(t)$ is the impulse response of the $k$-th user given in (0.6). The received signature waveform $g_k(t)$, in contrast to the transmitted signature waveform $g_0^k(t)$. The pulse response of the front-end filter (used in forming $g_k(t)$) is denoted by $y^{(k)}(t) = p(t) * h^{(k)}(t)$. 
When $h^{(k)}(t) = \delta(t), \forall k$, (0.44) reduces to the conventional case [18-21] and the conventional RAKE is thus reached. In simulations the estimated channel impulse response, $\hat{h}^{(k)}(t)$, will be used to replace the $h^{(k)}(t)$. With pulse distortion included in $\hat{h}^{(k)}(t)$, we call our new receiver structure (in Fig.0.3.5) the generalized RAKE structure.

The optimum receiver is defined as the receiver that selects the most probable sequence of bits $\{b_k(n), 0 \leq n \leq N, 1 \leq k \leq K\}$ given the received signal $r(t)$ observed over the time interval $0 \leq t \leq NT + 2T$.

The cross-correlation between pairs of signature waveforms play an important role in the metrics for the signal detector and on the performance. The pulse distortion affects the system through the cross-correlation. We define, where $\rho_{ij}(\tau) = \frac{1}{T} \int_0^T g_i(t)g_j(t+\tau)dt$, $\rho_{ij}(\tau) = \frac{1}{T} \int_0^T g_i(t)g_j(t+T+\tau)dt$ (0.45)

Similarly, we may define

$$
\rho_{ij}^0(\tau) = \frac{1}{T} \int_0^T g^0_i(t)g^0_j(t+\tau)dt, \quad \rho_{ij}^0(\tau) = \frac{1}{T} \int_0^T g^0_i(t)g^0_j(t+T+\tau)dt
$$

(0.46)

It is important to connect these cross-correlation functions through the channel impulse response via (0.44). As a result, we obtain

$$
\rho_{ij}(\tau) = \rho_{ij}^0(\tau) * [h^{(i)}(t) * h^{(j)}(-t)]
$$

(0.47)

When $h^{(i)}(t) = h^{(j)}(t)$, (0.47) reduces to the familiar auto-correlation form. Further when $h^{(i)}(t)$ and $h^{(j)}(t)$ can be modeled by the Turin’s model, (0.42) reduces to $r(t) = x(t) + n(t)$, which is the conventional form in [18].

Let us consider the synchronous transmission. In AWGN, it is sufficient to consider the signal received in one signal interval, say $0 \leq t \leq T$, and determine the optimum receiver. The optimum maximum-likelihood receiver is based on the log-likelihood function

$$
\Lambda(b) = \int_0^T \left[ r(t) - \sum_{k=1}^K \sqrt{E_k} b_k(1) g_k(t) \right]^2 dt
$$

(0.48)

and selects the information sequence $\{b_k(1), 1 \leq k \leq K\}$ that minimizes $\Lambda(b)$. Expanding (0.48) leads to

$$
\Lambda(b) = \int_0^T r^2(t)dt - 2 \sum_{k=1}^K \sqrt{E_k} b_k(1) \int_0^T r(t)g_k(t)dt + \sum_{j=1}^K \sum_{k=1}^K \sqrt{E_j E_k} b_j(1) b_k(1) \int_0^T g_j(t)g_k(t)dt
$$

(0.49)

The term $r_k = \int_0^T r(t)g_k(t)dt$ represents the cross-correlation of the received signal with the k-th signature. In practice, through (0.44), $\hat{g}_k(t) = g^0_k(t) * \hat{h}^{(k)}(t)$ is used where $\hat{h}^{(k)}(t)$ is the estimated channel impulse response. The equation (0.49) may
be expressed compactly as
\[ \Omega(r_K, b_K) = 2b_K^t r_K - b_K^t R_s b_K \]  
(0.50)

where
\[ r_K = [r_1, r_2, \cdots, r_K]^t \]
\[ b_K = [\sqrt{E_1} b_1(1), \cdots, \sqrt{E_K} b_K(1)]^t \]
and the prime denotes matrix transpose, \( R_s \) is the correlation matrix with elements \( \rho_{jk}(0) \). Pulse distortion enters the statistic through \( \rho_{jk}(0) = \int_0^T g_j(t)g_k(t) \, dt \).

The received signal vector \( r_K \) represents the output of \( K \) correlators or matched filters—one for each of the \( K \) signatures. In the generalized RAKE structure these matched filters matches the shapes of the received distorted multipath pulses, instead of the shapes of the transmitted pulses. The \( r_K \) is given by
\[ r_K = R_s b_K + n_K \]  
(0.51)

where
\[ b_K(i) = [\sqrt{E_1} b_1(i), \sqrt{E_2} b_2(i), \cdots, \sqrt{E_K} b_K(i)]^t \]
\[ n_K = [n^T(1) \cdots n^T(N)]^t \]

The noise vector \( n_K \) has a covariance
\[ E \left[ n_K n_K^t \right] = \frac{1}{2} N_0 R_s \]  
(0.52)

where the prime denotes matrix transpose.

The rest of detection follows the standard steps in [4,17,18]. The channel estimation is addressed in the section of generalized RAKE receiver.

REFERENCES


