Wideband Waveform Optimization with Energy Detector Receiver in Cognitive Radio

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Abstract—This paper investigates the transmitted waveform optimization issues for wideband cognitive radio with energy detector receiver. The motivation is to provide a cheap cognitive radio network with simple and cheap cognitive radio receivers. In cognitive radio, a spectral mask for the transmitted waveform is determined upon spectrum sensing, and arbitrary transmitted spectral shaping is required. Meanwhile, the interferences from primary radio should be notched at the secondary receivers. The contribution of this paper is to optimize the cognitive radio communication link by jointly considering the optimization objective, the spectral constraint at the transmitter and the interference cancellation at the receiver. The optimization objective considered here is to maximize captured signal energy within integration window. The optimization problem is formulated as the Quadratically Constrained Quadratic Program (QCQP), which is known as an NP-hard problem. However, after relaxation, QCQP can be solved by using Semidefinite Program (SDP) plus a randomized algorithm. In addition to SDP, a computationally-efficient iterative method is proposed to give the optimal solution for a special situation where the integrator only captures one sample as the decision statistic.

I. INTRODUCTION

Waveform design or optimization is a key research issue in the current wireless communication system. Waveform should be designed according to different requirements and objectives of system performance. For example, the waveform can be designed to carry more information to the receiver in terms of capacity. For navigation and geolocation, ultra short waveforms should be used to increase the resolution. If an energy detector is employed at the receiver, the waveform should be optimized such that the energy of the signal within integration window at the receiver is maximized. In the context of cognitive radio, waveform design (or optimization) gives us more flexibility to design a radio which can coexist with other cognitive radios and primary radios. From cognitive radio’s point of view, spectral mask constraint at the transmitter and the influence of Arbitrary Notch Filter at the receiver should be seriously considered for waveform design or optimization, in addition to the considerations of the traditional communication objectives. Spectral mask constraint is imposed on the transmitted waveform such that cognitive radio generates no interference to the primary radio, while arbitrary notch filter is used at the secondary receiver to cancel the interference from primary radio. Although the arbitrary notch filter is placed at the receiver, the influence of arbitrary notch filter should be incorporated into transmit waveform design. How to implement the arbitrary notch filter is beyond the scope of this paper.

The optimization problem is formulated as the Quadratically Constrained Quadratic Program (QCQP), which is known as an NP-hard problem. However, after relaxation, QCQP can be solved by using Semidefinite Program (SDP) plus a randomized algorithm. In addition to SDP, a computationally-efficient iterative method is proposed to give the optimal solution for a special situation where the integrator only captures one sample as the decision statistic.

The rest of the paper is organized as follows. In Section II the system is described and how to get the optimal waveform without consideration of constraints is presented. Wideband waveform design for cognitive radio is studied in Section III. Numerical results are provided in Section IV, followed by some remarks given in Section V.

II. SYSTEM DESCRIPTION AND OPTIMAL WAVEFORM

The system architecture is shown in Figure 1. We limit our discussion to a single-user scenario, and consider the transmitted signal with OOK modulation given by

\[ s(t) = \sum_{j=-\infty}^{\infty} d_j p(t - j T_b) \]  

(1)

where \( T_b \) is the symbol duration, \( p(t) \) is the transmitted symbol waveform defined over \([0, T_p]\) and \( d_j \in \{0, 1\} \) is the \( j \)-th transmitted bit. Without loss of generality, assume the minimal propagation delay is equal to zero. The energy of \( p(t) \) is \( E_p \),

\[ \int_0^{T_p} p^2(t) \, dt = E_p \]  

(2)

and the frequency domain representation of \( p(t) \) is \( p_f(f) \). Thus,

\[ \int_{-\infty}^{\infty} |p_f(f)|^2 \, df = E_p \]  

(3)
The received noise-polluted signal at the output of low noise amplifier (LNA) is given by
\[ r(t) = h(t) \otimes s(t) + n(t) \]
and the frequency domain representation of \( h(t) \) is \( h(f) \). Thus,
\[ \int_{-\infty}^{\infty} |h(f)|^2 df = E_h \]  
(6)
where \( h(t), t \in [0, T_h] \) is the multipath impulse response that takes into account the effect of channel impulse response, the RF front-ends in the transceivers such as Power Amplifier, LNA and Arbitrary Notch Filter as well as antennas. \( h(t) \) is available at the transmitter [1][2] and the energy of \( h(t) \) is \( E_h \),
\[ \int_0^{T_h} h^2(t) dt = E_h \]  
(5)
and the frequency domain representation of \( h(t) \) is \( h(f) \).

\( \otimes \) denotes convolution operation. \( n(t) \) is a low-pass additive zero-mean Gaussian noise with one-sided bandwidth \( W \) and one-sided power spectral density \( N_0 \), \( x(t) \) is the received noiseless symbol-"1" waveform defined as
\[ x(t) = h(t) \otimes p(t) \]  
(7)
where \( h(t) \) is the starting time of integration for each symbol and \( 0 \leq T_{0} < T_{10} + T_{1}, \) i.e. no existence of ISI.

An energy detector performs square operation to \( r(t) \). Then the integrator does the integration over a given integration window \( T_1 \). Corresponding to the time index \( k \), the \( k \)-th decision statistic at the output of the integrator is given by
\[ z_k = \int_{kT_h+T_{10}+T_1}^{(k+1)T_h+T_1} r^2(t) dt \]
\[ = \int_{kT_h}^{(k+1)T_h+T_1} (d_kx(t-kT_h) + n(t))^2 dt \]  
(8)
where \( T_{10} \) is the starting time of integration for each symbol and \( 0 \leq T_{10} < T_0 + T_1 \leq T_x \leq T_h \).

An approximately equivalent SNR for the energy detector receiver, which provides the same detection performance when applied to a coherent receiver, is given as [3]
\[ \text{SNR}_{eq} = \frac{2 \left( \int_{T_{10}}^{T_{10}+T_1} x^2(t) dt \right)^2}{2.3T_1 W N_0^2 + N_0 \int_{T_{10}}^{T_{10}+T_1} x^2(t) dt} \]  
(9)
For best performance, the equivalent SNR \( \text{SNR}_{eq} \) should be maximized. Define,
\[ E_I = \int_{T_{10}}^{T_{10}+T_1} x^2(t) dt \]  
(10)
For given \( T_1, N_0 \) and \( W \), \( \text{SNR}_{eq} \) is the increasing function of \( E_I \). So the maximization of \( \text{SNR}_{eq} \) in Equation (9) is equivalent to the maximization of \( E_I \) in Equation (10).

So the optimization problem to get the optimal \( p(t) \) is shown below,
\[ \max \int_{T_{10}}^{T_{10}+T_1} x^2(t) dt \]
\[ \text{s.t.} \int_0^{T_h} p^2(t) dt = E_p \]  
(11)
In order to solve the optimization problem (11), a numerical approach is employed in this paper. In other words, \( p(t), h(t) \) and \( x(t) \) are uniformly sampled (assumed at Nyquist rate), and the optimization problem (11) will be converted to its corresponding discrete-time form. Assume the sampling period is \( T_s \), \( T_p/T_s = N_p \) and \( N_p \) is assumed to be even, \( T_h/T_s = N_h \) and \( T_x/T_s = N_x \). So \( N_x = N_p + N_h \).

\( p(t), h(t) \) and \( x(t) \) are represented by \( p_i, i = 0, 1, \ldots, N_p \), \( h_j, i = 0, 1, \ldots, N_h \) and \( x_i, i = 0, 1, \ldots, N_x \) respectively [3].

Define,
\[ p = [p_0 p_1 \cdots p_{N_p}]^T \]  
(12)
and
\[ x = [x_0 x_1 \cdots x_{N_x}]^T \]  
(13)
Construct channel matrix \( H_{(N_x+1) \times (N_p+1)} \),
\[ (H)_{i,j} = \begin{cases} h_{i-j}, & 0 \leq i - j \leq N_h \\ 0, & \text{else} \end{cases} \]  
(14)
where \( (\bullet)_{i,j} \) denotes the entry in the \( i \)-th row and \( j \)-th column of the matrix or vector. Meanwhile, for vector, taking \( p \) as an example, \( (p)_{i,1} \) is equivalent to \( p_{i-1} \).

The matrix expression of Equation (7) is,
\[ x = Hp \]  
(15)
and the constraint in the optimization problem (11) can be expressed as,
\[ \|p\|_2^2 = E_p \]  
(16)
where \( \|p\|_2^2 \) denotes the norm-2 of the vector. In order to make the whole paper consistent, we further assume,
\[ \|p\|_2^2 = 1 \]  
(17)
Let \( T_1/T_s = N_1 \) and \( T_{10}/T_s = N_{10} \). The entries in \( x \) within integration window constitute \( x_I \) as,
\[ x_I = [x_{N_{10}} x_{N_{10}+1} \cdots x_{N_{10}+N_1}]^T \]  
(18)
and \( E_I \) in Equation (10) can be equivalently shown as,
\[ E_I = \|x_I\|_2^2 T_s \]  
(19)
Simply dropping $T_i$ in $E_I$ will not affect the optimization objective, so $E_I$ is redefined as,

$$E_I = \|x_I\|^2_2$$  \hspace{1cm} (20)

Similar to Equation (15), $x_I$ can be obtained by,

$$x_I = H_I p$$  \hspace{1cm} (21)

where $(H_I)_{i,j} = (H)_{Ni+1,i,j}$ and $i = 1, 2, \ldots, N_I + 1$ as well as $j = 1, 2, \ldots, N_p + 1$.

The optimization problem (11) can be represented by its discrete-time form as,

$$\max E_I \quad \text{s.t.} \quad \|p\|^2_2 = 1$$  \hspace{1cm} (22)

The optimal solution $p^*$ for the optimization problem (22) is the dominant eigen-vector in the following eigen-function [3].

$$H_I^T H_I p = \lambda p$$  \hspace{1cm} (23)

Furthermore, $E_I^p$ will be obtained by Equation (20) and Equation (21).

III. WIDEBAND WAVEFORM DESIGN IN COGNITIVE RADIO

For cognitive radio, there is a spectral mask constraint for the transmitted waveform. Based on the previous discussion, $p$ is assumed to be the transmitted waveform, and $F$ is the discrete-time Fourier transform operator, thus the frequency domain representation of $p$ is,

$$p_f = Fp$$  \hspace{1cm} (24)

where $p_f$ is a complex value vector. If the $i$-th row of $F$ is $f_i$, then each complex value in $p_f$ can be represented by,

$$(p_f)_{i,1} = f_i p, i = 1, 2, \ldots, \frac{N_p}{2} + 1$$  \hspace{1cm} (25)

Define,

$$F_i = f_i^H f_i, i = 1, 2, \ldots, \frac{N_p}{2} + 1$$  \hspace{1cm} (26)

Given the spectral mask constraint in terms of power spectral density $c = \left[ c_1 c_2 \cdots c_{\frac{N_p}{2} + 1} \right]^T$, so

$$| (p_f)_{i,1} |^2 = | f_i p |^2 = p^T f_i^H f_i p = p^T F_i p \leq c_i, i = 1, 2, \ldots, \frac{N_p}{2} + 1$$  \hspace{1cm} (27)

where $| \cdot |$ is the modulus of the complex value.

So the optimization problem with spectral mask constraint can be expressed as,

$$\max E_I \quad \|p\|^2_2 \leq 1 \quad p^T F_i p \leq c_i$$  \hspace{1cm} (28)

s.t.

$$p^T F_{N_p+1} p \leq c_{N_p+1}$$

Define,

$$H_o = H_I^T H_I$$  \hspace{1cm} (29)

Equation (20), Equation (21) and Equation (29) are put into the optimization problem (28), and then this optimization problem will be formulated as,

$$\max p^T H_o p \quad p^T p \leq 1 \quad p^T F_i p \leq c_i$$  \hspace{1cm} (30)

s.t.

$$p^T F_{N_p+1} p \leq c_{N_p+1}$$

In the optimization problem (30), both the objective function and the constraints are quadratic functions, so this kind of optimization problem is a QCQP. The general QCQP is a NPhard problem. A semidefinite relaxation method is used in this paper to give the suboptimal solution to the optimization problem (30).

Define,

$$P = pp^T$$  \hspace{1cm} (31)

So $P$ should be the symmetric positive semidefinite matrix, i.e. $P \succ 0$ and rank of $P$ should be equal to 1.

Then,

$$E_I = p^T H_o p = \text{trace} (H_o pp^T) = \text{trace} (H_o P)$$  \hspace{1cm} (32)

and $\|p\|^2 = \text{trace} (P), p^T F_i p = \text{trace} (F_i P), i = 1, 2, \ldots, \frac{N_p}{2} + 1$.

Rank constraint is nonconvex constraint, so after dropping it, QCQP is relaxed to SDP,

$$\max \text{trace} (H_o P) \quad \text{trace} (P) \leq 1 \quad \text{trace} (F_i P) \leq c_i$$  \hspace{1cm} (33)

s.t.

$$\text{trace} \left( F_{N_p+1} P \right) \leq c_{N_p+1}$$

The optimal solution $P^*$ of the optimization problem (33) can be obtained by using CVX tool [4] and the value of the objective function gives the upper bound of the optimal value in the optimization problem (30). If the rank of $P^*$ is equal to 1, then the dominant eigen-vector of $P^*$ will be the optimal solution $p^*$ for the optimization problem (30). But if the rank of $P^*$ is not equal to 1, the dominant eigen-vector of $P^*$ can not be treated as the optimal solution for the optimization problem (30), because of the violation of bound constraint.

An efficient randomized algorithm is given by [5] [6] to obtain the feasible solution $p^*$ from $P^*$.

This algorithm will be repeated a sufficient number of times to get the relatively good feasible $p^*$ as the suboptimal solution to the optimization problem (30).

When $T_i$ approaches zero, which means the integrator only captures one sample after ADC as the decision statistic, the
maximization of $E_f$ will be equivalent to the maximization of $x(T_{f0})$. An iterative algorithm is proposed here to give the solution to the maximization of $x(T_{f0})$, which will be more computationally efficient than the previous semidefinite relaxation method. For the simplicity of the following presentation, $T_{f0}$ is assumed to be zero.

From the inverse Fourier transform,

$$\int x(t) = \int_{-\infty}^{\infty} x_f(f) e^{j2\pi ft} df$$

and from Equation (7),

$$x_f(f) = h_f(f)p_f(f)$$

where $x_f(f)$ is the frequency domain representation of $x(t)$ respectively.

So,

$$x(0) = \int_{-\infty}^{\infty} x_f(f) df = \int_{-\infty}^{\infty} h_f(f)p_f(f) df$$

If there is no spectral mask constraint, then according to the Cauchy–Schwarz inequality,

$$x(0) = \sqrt{\int_{-\infty}^{\infty} |h_f(f)|^2 df \int_{-\infty}^{\infty} |p_f(f)|^2 df}$$

when $p_f(f) = \beta h_f^*(f)$ for all $f$, the equality is obtained. In this case, $p(t) = \beta h(-t)$, which means the optimal waveform is the time reversed multipath impulse response. And,

$$\beta = \sqrt{\frac{E_p}{E_h}}$$

If there is a spectral mask constraint, then the following optimization problem will become more complicated.

$$\max x(0)$$

s.t. $\int_{-\infty}^{\infty} |p_f(f)|^2 df \leq 1$

$$|p_f(f)|^2 \leq c_f(f)$$

where $c_f(f)$ represents the arbitrary spectral mask constraint.

Because $p_f(f)$ is the complex value, the phase and the modulus of $p_f(f)$ should be determined.

$$x(0) = \int_{-\infty}^{\infty} h_f(f)p_f(f) df$$

$$x(0) = \int_{-\infty}^{\infty} |h_f(f)|^2 |p_f(f)|^2 df$$

where the angular component of the complex value is $\arg (\bullet)$. For the real value signal $x(t)$,

$$x_f(f) = x_f^*(-f)$$

where “*” denotes conjugate operation.

$$x_f(f) = |h_f(f)||p_f(f)| e^{j2\pi(\arg(h_f(f)) + \arg(p_f(f)))}$$

and $x_f(f) + x_f(-f)$ is equal to

$$|h_f(f)||p_f(f)| \cos(2\pi (\arg(h_f(f)) + \arg(p_f(f))))$$

If $h(f)$ and $|p_f(f)|$ is given, maximization of $x(0)$ is equivalent to setting,

$$\arg(h(f)) + \arg(p(f)) = 0$$

which means the angular component of $p_f(f)$ is the negative angular component of $h_f(f)$.

The optimization problem (39) can be simplified as,

$$\max \int_{-\infty}^{\infty} |h_f(f)||p_f(f)| df$$

s.t. $\int_{-\infty}^{\infty} |p_f(f)|^2 df \leq 1$

$$|p_f(f)|^2 \leq c_f(f), f \geq 0$$

Because,

$$|h_f(f)| = |h_f(-f)|$$

$$|p_f(f)| = |p_f(-f)|$$

$$|c_f(f)| = |c_f(-f)|$$

for all $f$. Thus uniformly discrete frequency points $f_0, \ldots, f_M$ are considered in the optimization problem (46). Meanwhile, $f_0$ corresponds to the DC component and $f_1, \ldots, f_M$ correspond to the positive frequency component.

Define vector $h_f$,

$$(h_f)_{i,1} = \begin{cases} \sqrt{2} |h_f(f_{i-1})|, & i = 1 \\ \sqrt{2} |h_f(f_{i-1})|, & i = 2, \ldots, \frac{N_p}{2} + 1 \end{cases}$$

Define vector $p_f$,

$$(p_f)_{i,1} = \begin{cases} \sqrt{2} |p_f(f_{i-1})|, & i = 1 \\ \sqrt{2} |p_f(f_{i-1})|, & i = 2, \ldots, \frac{N_p}{2} + 1 \end{cases}$$

Define vector $c_f$,

$$(c_f)_{i,1} = \begin{cases} \sqrt{2} c_f(f_{i-1}), & i = 1 \\ \sqrt{2} c_f(f_{i-1}), & i = 2, \ldots, \frac{N_p}{2} + 1 \end{cases}$$

Thus, the discrete version of the optimization problem (46) is shown below,

$$\max h_f^T p_f$$

s.t. $||p_f||_2^2 \leq 1$

$$P_f \leq c_f$$

So an iterative algorithm is proposed to get the optimal solution $p_f^*$ to the optimization problem (53) as follows.

1. Initialization: $P = 1$ and $p_f^*$ is set to be an all-0 column vector.

2. Solve the following optimization problem to get the optimal $q_f^*$ using Cauchy–Schwarz inequality.

$$\max h_f^T q_f$$

s.t. $||q_f||_2^2 \leq P$
3. Find $i$, such that $(q_f^*)_{i,1}$ is the maximal value in the set $\{ (q_f^*)_{j,1} \mid (q_f^*)_{j,1} > (c_f)_{j,1} \}$. If $\{i\} = \emptyset$, then the algorithm is terminated and $p_f^* := p_f^* + q_f^*$. Otherwise go to step 4.

4. Set $(p_f^*)_{i,1} = (c_f)_{i,1}$.

5. $P := P - (c_f)_{i,1}^2$ and set $(h_f)_{i,1}$ to zero. Go to step 2.

When $p_f^*$ is obtained for the optimization problem (53), from Equation (45) and Equation (51), the optimal $p_f(f)$ and the corresponding $p(t)$ can be smoothly achieved.

### IV. Numerical Results

The following setting has been used in generating numerical results: $T_s = 0.5\text{ns}$, $T_h = 100\text{ns}$, $T_p = 100\text{ns}$ and $T_{I0} + \frac{T_I}{2} = 100\text{ns}$. The transfer function of the multipath impulse response $h(t)$ obtained from measured data and a notched filter, as shown in Figure 2, is considered. In the following presentation three types of designed waveforms will be mentioned: (1) Waveform SDP Unverified, obtained by SDP and the rank of $P^*$ to the optimization problem (33) is not equal to 1; (2) Waveform SDP, obtained by SDP and the rank of $P^*$ to the optimization problem (33) is equal to 1, or obtained by SDP combined with a randomized algorithm; and (3) Waveform by Iterative Algorithm, obtained by an iterative algorithm.

Fig. 3 and Fig. 4 show the simulation results when the integration window size considered here is $T_I \rightarrow 0$, which means the integrator only captures one sample as the decision statistic. In this case, both the iterative algorithm and the SDP method yield the same waveform. However, the iterative algorithm is more computationally efficient. SDP in this kind of situation always gives the optimal solution to the optimization problem (30), which means the rank of $P^*$ to the optimization problem (33) is equal to 1. It can be seen that the designed waveforms satisfy the spectral mask constraint thus do not interfere with others, and contain minimized energies in the notched bands.

Spectral results for $T_I = 20\text{ ns}$ with two different spectral mask constraints are plotted in Figures 5 and Figure 6. When $T_I$ is greater than 0, SDP on its own cannot always guarantees a feasible solution to the original problem, or equivalently, the rank of $P^*$ to the optimization problem (33) is not always equal to 1. In the case of spectral mask 2, SDP itself gives the optimal solution; but in the case of spectral mask 3, the waveform obtained by only SDP violates the spectral mask constraints in some frequency points, and SDP combined with the randomized algorithm gives the suboptimal solution.

### V. Conclusion

This paper deals with wideband waveform optimization with energy detector in cognitive radio. Wideband waveform is designed according to the optimization objective with the considerations of the spectral constraint at the transmitter and the influence of arbitrary notch filter at the receiver. The optimization problem is formulated as QCQP. After relaxation, QCQP can be solved by using SDP plus a randomized algorithm. Meanwhile, a computationally-efficient iterative method is proposed to give the optimal solution for a special situation where the integrator only captures one sample as the decision statistic.
Fig. 5. Spectral mask 2, notch line of Arbitrary Notch Filter, the designed waveforms and multipath impulse response represented in the frequency domain when $T_I$ is 20 ns.

Fig. 6. Spectral mask 3, notch line of Arbitrary Notch Filter, the designed waveforms and multipath impulse response represented in the frequency domain when $T_I$ is 20 ns.

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